BINDURA UNIVERSITY OF SCIENCE EDUCATION

SFM111/AMT101/ MT101: CALCULUS 1

CALCULUS AND FINANCIAL MODELING

NOV 2024

Time : 3 hours

Answer ALL questions in Section A and at most TWO questions in section B.

SECTION A (40 marks)

Candidates may attempt ALL questions being careful to number them A1 to A4.

A1. (a) Find all critical points for $f(x) = x^3 - 3x^2 + 1$. [7]

(b) Find all the local maximumm and minimum points for $f(x) = \frac{1}{2} + sin(x)$ on $[0, \pi]$. [7]

A2. Sole the following inequalities, $|5x - 8| \leq 12$. [4]

A3. (a) State the $\varepsilon - N$ definition of the limit of a sequence. [2]

(b) Prove that $lim_{n\to\infty}(\frac{n-1}{n+1}) = 1$. [5]

A4. Find the derivativve of $sin^{-1}(x)$. [5]

A5. (a) Show that the function $f(x) = 5x - 1$ is bijective. [6]

(b) Find the inverse of $f(x) = 5x - 1$. [4]

SECTION B (60 marks)

Candidates may attempt TWO questions being careful to number them B5 to B7.

B6. (a) Evaluate the following limits.

(i) $lim_{x\to\infty}[x^4 sin^4(\frac{1}{x})]$. [5]

(ii) $lim_{x\to 0}[x^2 sin(\frac{1}{x})]$. [5]

(b) Find the indefinite integral of $\int \frac{x^3+2}{x^3-x}dx$. [8]

(c) Use the $\epsilon - N$ definition of the limit of a sequence to show that a sequence whose n^{th} term given by $a_n = (3 - \frac{1}{7n^2})$ converges to 3. [6]

(d) Show that the sequence $U_n = \dfrac{2n-7}{3n+2}$ is monotonic increasing. [6]

B7. (a) When do we say a sequence S_n is convergent. [2]

(b) Determine whether the sequence $S_n = \dfrac{(-1)^{n+1}(n^2)}{n^2+1}$, where $n = 1, 2, 3..$ converges or diverges. [4]

(c) Find Df and Rf if $f(x) = \dfrac{1}{\sqrt{3-x}}$. [4]

(d) Let $f(x) = x^2 sin(\dfrac{1}{x}), x \neq 0$.

(i) Does $f(x)$ have a derivative at $x = 0$? Justify your answer. [5]

(ii) Is $f(x)$ differentiable at $x = 0$, justify your answer. [5]

(e) Find the dimensions of an isosceles triangle of largest area that can be inscribed in a circle of radius a units. [10]

B8. (a) State the Mean Value Theorem of differentiation. [3]

(b) Verify the Mean Value Theorem for $f(x) = x^2$ where $a = 0$ and $b = 1$. [7]

(c) State the second fundamental theorem of calculus. [2]

(d) Find the area of the region bounded by $f(x) = 4 - 4x^2$ and $g(x) = 1 - x^2$. [8]

(e) Give a detailed sketch of the graph of $y = \dfrac{x^3}{3x-2}$. [10]

END OF QUESTION PAPER