BINDURA UNIVERSITY OF SCIENCE EDUCATION MT102

BACHELOR OF SCIENCE EDUCATION

LINEAR MATHEMATICS 1

AUG 2023

Time: 3 Hours

Candidates may attempt ALL questions in Section A and at most TWO questions in Section B.

Each question should start on a fresh page.

SECTION A (40 marks)

A1. Define the following terms:

(i) Symmetric matrix. [1]

(ii) Trace of a matrix. [2]

A2. Write the following expressions (2+3i)(-2-3i) in the form $a + bi$. [3]

A3. (a) Define the cross product of two vectors $\boldsymbol{u}$ and $\boldsymbol{v}$. [2]

(b) Given that $\boldsymbol{u} = 2\boldsymbol{i} - 2\boldsymbol{j} + 2\boldsymbol{k}$ and $\boldsymbol{v} = 3\boldsymbol{i} + 2\boldsymbol{j} - 2\boldsymbol{k}$, find $\boldsymbol{u} \times \boldsymbol{v}$ and a unit vector perpendicular to the plane containing the vectors $\boldsymbol{u}$ and $\boldsymbol{v}$. [5]

A4. Let $A = \begin{bmatrix} 3 & 2 & -1 \\ 1 & 6 & 3 \\ 2 & -4 & 0 \end{bmatrix}$.

(a) Find the adjoint of the matrix A. [5]

(b) Hence or otherwise find A^{-1}, the inverse of matrix A. [2]

A5. (a) Find the solution of the following system of linear equations using Gauss elimination method

$$x_1 + x_2 - 2x_3 = -6$$
$$2x_1 - x_2 - x_3 = 3$$
$$-2x_1 + 2x_2 + 3x_3 = -2.$$ [6]

(b) For the matrices $A = \begin{bmatrix} 1 & 2 \\ 2 & 0 \\ -1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -1 & 2 \\ 2 & 1 & 0 \end{bmatrix}$ verify that $(AB)^t = B^t A^t$. [6]

A6. (a) For any vectors **a** and **b** prove the identity $|\mathbf{a} \times \boldsymbol{b}|^2 + |\boldsymbol{a}.\boldsymbol{b}|^2 = |\boldsymbol{a}|^2 |\boldsymbol{b}|^2$. [4]

(b) Find x such that $det = 0$ if $A = \begin{bmatrix} 1 & x & x \\ -x & -2 & x \\ x & x & 3 \end{bmatrix}$. [4]

SECTION B (60 marks)

Candidates may attempt TWO questions being careful to number them B7 to B9.

B7. (a) Find the solution to the linear system of equations,

$$x_1 + x_2 + 2x_3 = 9$$
$$2x_1 + 4x_2 - 3x_3 = 1$$
$$3x_1 + 6x_2 - 5x_3 = 0$$

using Crammers rule. [8]

(b) Find the values of k for which the following system of linear equations,

$$x_1 + x_2 = -2$$
$$2x_1 + 4x_2 = 3$$
$$3x_1 + 6x_2 = k \quad \text{is consistent.}$$ [4]

(c) Let $A = \begin{bmatrix} 1 & 3 & 0 \\ 4 & -1 & 2 \end{bmatrix}$, calculate A^tA. [4]

(d) Solve the following system of simultaneous equations using Gauss elimination,

$$x_1 + x_2 + 2x_3 = 1$$
$$2x_1 - x_2 + x_4 = -2$$
$$x_1 - x_2 - x_3 - 2x_4 = 4$$
$$2x_1 - x_2 + 2x_3 + x_4 = 0.$$ [10]

(e) Find the area of a triangle with the end points (1, 2, 1) (4,3,2) and (4, 1, 4). [4]

B8. (a) Find the equation of a plane containing the points: P(2, 1, 1), Q(0, 4, 1) and R(-2, 1, 4). [8]

(b) Find the area of the parallelogram with the vectors $\boldsymbol{u} = < 2, 3, 4 >$ and $\boldsymbol{w} =< 6, 1, 4 >$. [4]

(c) If $\mathbf{v} = 2\mathbf{i} - \mathbf{j} + \mathbf{k}$ and $\mathrm{u} = \mathbf{i} + 2\mathbf{j} - 3\mathbf{k}$, Find $|(2\mathbf{v} + \mathbf{u}) \times (\mathbf{v} - 2\mathbf{u})|$. [5]

(d) Given that $\boldsymbol{a} = 3\boldsymbol{i} - 2\boldsymbol{j} + 6\boldsymbol{k}$ and $\boldsymbol{b} = 2\boldsymbol{i} + 3\boldsymbol{j} - 4\boldsymbol{k}$, find $\boldsymbol{a} \times \boldsymbol{b}$. [3]

(e)(i) Find the cartesian equation of a line in space which passes through the points P(1; 0; −1) and Q(3; −2; −3). [4]

(ii) Find the angle between the planes given by $x - 2y + z = 0$ and $2x + 3y - 2z = 0$ and find the parametric equations for the line of intersection. [6]

B9. (a) Solve the following equation:

$x^4 - 3x^3 + 11x^2 + 5x + 26 = 0$, given that $2 + 3i$ is a root of the equation. [6]

(b) (i) Use Euler's formula to show that $\cos\theta = \frac{1}{2}(e^{i\theta} + e^{-i\theta})$ and $\sin\theta = \frac{1}{2i}(e^{i\theta} - e^{-i\theta})$. [5]

(ii) Hence show that $sin^4\theta = \frac{1}{8}cos4\theta - \frac{1}{2}cos2\theta + \frac{3}{8}$. [6]

(c) (i) State De Moivres's formula. [2]

(ii) Prove the identity $cos5\theta = 16cos^5\theta - 20cos^3\theta + 5cos\theta$. [6]

(d) Given that A= $\begin{bmatrix} 2 & -5 & -1 & 4 \\ -3 & 2 & 0 & 3 \\ 1 & 3 & 2 & 1 \\ 4 & 0 & 6 & -2 \end{bmatrix}$, find the determinant of A. [5]

END OF QUESTION PAPER