BINDURA UNIVERISTY OF SCIENCE EDUCATION

FACULTY OF SCIENCE

PHYSICS AND ENGINEERING DEPARTMENT

BACHELOR OF SCIENCE PHYSICS

HPH 211/PH 205: VECTOR METHODS AND ELECTROMAGNETISM

AUG 2024

DURATION: THREE HOURS

INSTRUCTIONS

*Answer **ALL** parts of Section A and any **THREE** questions from Section B. Section A carries 40 marks and Section B carries 60 marks.*

SECTION A

1(a) A plane wave solution to Maxwell's equations in a linear homogeneous dielectric is

Given $\mathbf{E}(z,t) = \mathbf{E}_0 \cos(\sqrt{6}\, z - 6 \times 10^{10}\, t)$,

where t is in seconds, z is in centimeters, and $\mathbf{E}_0$ is a constant vector.

(i) What part of the electromagnetic spectrum is this wave (radio, microwave, x-ray,etc.)? [2]

(ii) What is the index of refraction of the medium? [3]

(iii) What would the wavelength be if this wave traveled in free space? [3]

(iv) Give an expression for the associated magnetic field. [4]

(b) At the upper surface of the Earth's atmosphere, the time-averaged magnitude of the Poynting vector, $\langle S \rangle = 1.35 \times 10^3 W/m^2$, is referred to as the solar constant.

(i) Assuming that the Sun's electromagnetic radiation is a plane sinusoidal wave, what are the magnitudes of the electric and magnetic fields? [3]

(ii) What is the total time-averaged power radiated by the Sun? The mean Sun-Earth distance is

$R = 1.50 \times 10^{11} m$. [4]

(c) Prove that the speed of electromagnetic waves in vacuum is equal to the speed of light [3]

(d) Compare sound waves with electromagnetic waves. [3]

(e) A light bulb puts out 100 W of electromagnetic radiation. What is the time-average intensity of radiation from this light bulb at a distance of one meter from the bulb? What are the maximum values of electric and magnetic fields, and E_0 and B_0, at this same distance from the bulb? Assume a plane wave. [6]

(f) Consider a collection of N static point charges q_i, located at position vectors $\mathbf{r}_i$, respectively (where i runs from 1 to N). Determine the electrostatic energy stored in such a collection. In other words, calculate the amount of work required to assemble the charges, starting from an initial state in which they are all at rest and very widely separated.
[9]

SECTION B

(Answer **three** questions in this section.)

2 **(a)** Verify that, for $\omega = kc$,

$$E(x,t) = E_0 \cos(kx - \omega t)$$
$$B(x,t) = B_0 \cos(kx - \omega t)$$

Satisfy the one-dimensional wave equation

$$\left(\frac{\partial^2}{\partial x^2} - \frac{1}{c^2}\frac{1}{\partial t^2}\right)\begin{Bmatrix} E(x,t) \\ B(x,t) \end{Bmatrix} = 0$$ [10]

(b) A parallel-plate capacitor with circular plates of radius R and separated by a distance h is charged through a straight wire carrying current I, as shown in the Figure 2.0:

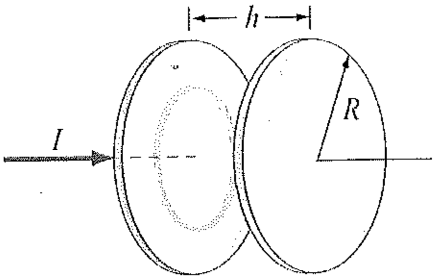

Fig 2.0 Parallel-plate capacitor

(i) Show that as the capacitor is being charged, the Poynting vector **S** points radially inward toward the center of the capacitor.

(ii) By integrating **S** over the cylindrical boundary, show that the rate at which energy enters the capacitor is equal to the rate at which electrostatic energy is being stored in the electric field. [10]

3 **(a)** A cylindrical conductor of radius a and conductivity σ carries a steady current I which is distributed uniformly over its cross-section, as shown in Figure 3.0

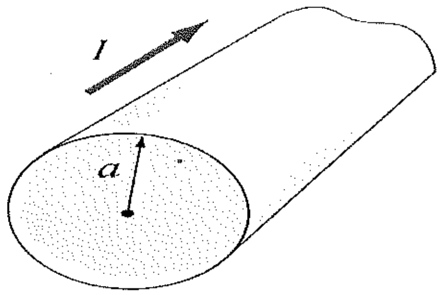

Fig 3.0 Cylindrical conductor

(i) Compute the electric field inside the conductor. **E**

(ii) Compute the magnetic field **B** just outside the conductor.

(iii) Compute the Poynting vector **S** at the surface of the conductor. In which direction does **S** point?

(v) By integrating **S** over the surface area of the conductor, show that the rate at which electromagnetic energy enters the surface of the conductor is equal to the rate at which energy is dissipated. [10]

(b) Explain in detail how standing Electromagnetic waves can be formed as shown below in Fig 3.1

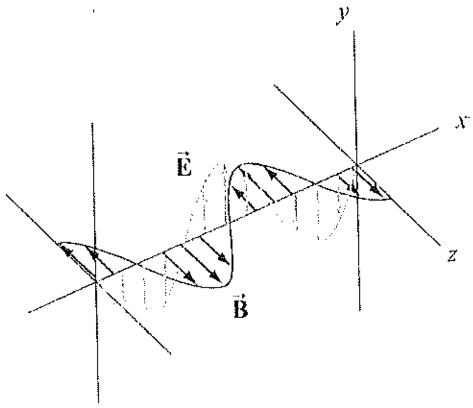

Fig 3.1 Standing Electromagnetic waves

[10]

4 **(a)** List the Maxwell`s equations in their differential and integral form and give their physical meanings [10]

(b) (i) Obtain the relationship between E_o and B_o

(ii) What causes electromagnetic radiation [10]

5(a) Consider a plane electromagnetic wave with the electric and magnetic fields given by

$$\vec{E}(x,t) = E_z(x,t)\hat{k}, \qquad \vec{B}(x,t) = B_y(x,t)\hat{j}$$

Show that the fields satisfy the following relationships

$$\frac{\partial E_z}{\partial x} = \frac{\partial B_y}{\partial t}, \qquad \frac{\partial B_y}{\partial x} = \mu_o \varepsilon_o \frac{\partial E_z}{\partial t}$$ [10]

(b) Derive the generalized Poynting Theorem for electromagnetic fields from Maxwell's equations in a linear dielectric:

$$\nabla \circ S + \frac{\partial U}{\partial t} = -\vec{J}_{free} \circ \vec{E}$$

Where $\vec{S} = \vec{E} \times \vec{H}$ is the Poynting vector and $U = \frac{1}{2}(\vec{E} \times \vec{D} + \vec{B} \times \vec{H})$ is the total energy density in the electromagnetic field. Note that the ohmic heating is due to the *free* current

6 **(a)** From Gauss's law, we know that this surface charge will give rise to a static electric field :

$$\vec{E}_0 = \begin{cases} +\left(\sigma/2\varepsilon_0\right)\hat{i}\, x > 0 \\ -\left(\sigma/2\varepsilon_0\right)\hat{i} < 0 \end{cases}$$

Now, at $t=0$, we grab the sheet of charge and start pulling it downward with constant velocity $\hat{}v = -v\mathbf{j}$. Examine and draw a diagram on how things will then appear at a later time t. In particular, before the sheet starts moving, look at the field line that goes through and after. [10]

(b) (i) List five conditions that satisfy the light propagation in uniform optical medium

(ii) show that the wave equations take the form

$$\nabla^2 \vec{E} = \mu\varepsilon \frac{\partial^2 \vec{E}}{\partial t^2}, \qquad \nabla^2 B = \mu\varepsilon \frac{\partial^2 \vec{B}}{\partial t^2}$$ [10]