BINDURA UNIVERSITY OF SCIENCE EDUCATION

MT009: PURE MATHEMATICS 3

Time : 3 hours

NOV 2024

Answer ALL questions in Section A and at most TWO questions in section B.

SECTION A (40 marks)

Candidates may attempt ALL questions being careful to number them A1 to A4.

A1. (a) Use Maclaurin's theorem to expand $ln(1+x)$ in ascending powers of x as far as the term in x^5. [5]

(b) Use Simpson's rule with six strips to estimate $\int_1^4 \sqrt{1+x^3}dx$. [8]

A2. (a) Use the trapezium rule to evaluate $\int_3^8 x^2 dx$ using strip width of one unit. [8]

(b) Find the vector product of $p = 3i - 4j + 2k$ and $q = 2i + 5j - k$. [5]

A3. Find the equation of the tangent to $f(x) = x^3 - 3x^2 + x - 1$ at the point where $x = 2$. [7]

A4. Find the equation of the normal to the curve $y = x^2 + 4x + 3$ at the point $(-1, 0)$. [7]

SECTION B (60 marks)

Candidates may attempt TWO questions being careful to number them B5 to B7.

B5. (a) Write down the unit vector which is perpendicular to the plane $2x + 3y + 6z = 10$. [3]

(b) Find the equation of the plane through the point $(1, 2, 3)$ and perpendicular to vector $4i + 5j + 6k$. [4]

(c) Find the angle between the plane $4x + 3y + 12z = 10$ and $8x - 6y = 14$. [8]

(d) Solve the differential equation $\frac{dy}{dt} = ky$ where $k > 0$. [5]

(e) What is the volume V of the cone swept out by the line $y = 2x$ rotated about the x-axis between $x = 0$ and $x = 5$. [5]

(f) Find the area included between the curves $y = \frac{1}{2}x^2$ and $y = \frac{7}{4}x$ from $x = 0$ to $x = 3, 5$. [5]

B6. (a) Find the angle between the vectors $V = 2i - 3j + k$ and $W = 6i + j - 2k$. [5]

(b) Find a vector orthogonal to both $V = 2i - 3j + k$ and $W = 6i + j - 2k$. [6]

(c) Use the Newton-Raphson method to solve $cos(x) = 2x$ correct to five decimal places. [6]

(d) Solve the deferential equation $(2x^2 + x)\frac{dy}{dx} + (8x + 4)y = 6x^2$. [13]

B7. (a) Derive the trapezium rule using five ordinates. [5]

(b) The equation of a curve is $2x^2 + 8xy + 5y^2 = -3$. Find the equations of two tangents which are parallel to the x-axis. [10]

(c) Find the positive integer n such that the cubic equation $x^3 - 9x - 12 = 0$ has a root between n and $n + 1$. Use linear interpolation once to find an approximation to this root. [8]

(d) O is the origin and A is the point on the curve $y = tan(x)$ where $x = \frac{\Pi}{3}$. The region S is enclosed by the arc OA, the y-axis and the line $y = \sqrt{3}$. Find the volume of the solid of revolution formed when S is rotated 360° about the x-axis, giving your answer in an exact form. [7]

END OF QUESTION PAPER