BINDURA UNIVERSITY OF SCIENCE EDUCATION

HONOURS DEGREE IN SCIENCE EDUCATION (HBScED) AUG 2024

MT303: Probability Theory and Statistics

Time: 3 hours

Candidates may attempts ALL questions in Section A and at most TWO questions in Section B. Each question should start on fresh page.

SECTION A (40 marks)

Candidate may attempt ALL questions being careful to number them A1 to A5

A1. (a) Define Singleton [2]

(b) Let X and Y be events and let $X \subset Y$. Show that $P(X) < P(Y)$. [4]

A2. Suppose A and B are independent events, prove that

(a) A' and B' are independent [3]

(b) $P(A'|B) = P(A')$. [3]

A3. (a) How many different permutations of the letters of the word MATHEMATICS are possible? [3]

(b) State the two properties of the legitimacy of a probability mass function, $p(x)$. [2]

(c) State the Uniqueness Theorem of the moment generating theorem. [3]

A4. Let X have the probability density function is given by:

$$f(x) = 2^{-|x-1|-1} \qquad for\ x = 0, 1, 2$$

(a) Determine the probability distribution of X in tabular form. [3]

(b) Find E(X) and Var(X). [4]

(c) Find the cumulative distribution function of X. [3]

A5. (i) Prove the property of memoryless of the exponential random variable. [5]

(ii) If EX(X-1)=4 for an exponential random variable X, find the value of λ. [5]

SECTION B (60 Marks)

Candidates may attempt TWO questions being careful to number them B6 to B8.

B6. (a) Let X have the probability density function is given by:

$$f_X(x) = \begin{cases} 2x & 0 \le x \le b \\ 0 & otherwise \end{cases}$$

(i) Sketch the graph of $f_X(x)$. [3]

(ii) Find and sketch the cumulative frequency of X. [5]

(iii)Hence, find $P(0 < X < \frac{1}{2})$. [4]

(b) Let X be a random variable with probability mass function given by:

$$p(x) = \begin{cases} \theta(1-\theta)^{x-1} & if\ x = 1, 2, 3, \ldots \\ 0 & otherwise \end{cases}$$

By differentiating with respect to θ both sides of the equation

$$\sum_{x=1}^{\infty} \theta(1-\theta)^{x-1} = 1$$

Show that the mean of the geometric distribution is given by $\frac{1}{\theta}$. [6]

(c) State and prove Bayes theorem. [12]

B7. (a) State and prove the Chebyshev's inequality. [12]

(b) If $X \sim B(n, p)$

(i) Find the moment generating function of X. [4]

(ii) Hence find E(X) and Var(X). [4, 4]

(c) State and prove the Law of total probability [6]

B8. (a) Let X be a continuous random variable with parameter λ and probability density function given by:

$$f_X(x) = \lambda e^{-\lambda x}, \qquad x > 0.\ \lambda > 0$$

Show that for any positive number s and t, $P(X > s + t | X > s) = P(X > t)$. [10]

(b) Show that the moment generating function of the normal distribution is given by:

$$M_X(t) = e^{\mu t + \frac{1}{2}\sigma^2 t^2}.$$ [5]

(c) Let $\psi = (-\infty; \infty)$ be the universal set.
Use De Morgan's rule to find $([0,3]\ [1,5])^c$. [5]

(d) State and prove the Bayes' theorem. [10]

END OF THE PAPER