BINDURA UNIVERSITY OF SCIENCE EDUCATION MT005

DIPLOMA IN SCIENCE EDUCATION PART 2

PURE MATHEMATICS 2

Time : 2 hours NOV 2024

Candidates may attempt ALL questions in Section A and at most TWO questions in Section B. Each question should start on a fresh page.

SECTION A (40 marks)

Candidates may attempt ALL questions being careful to number them A1 to A5.

A1. (a) Find the equation of the line through $(0;2)$ and parallel to the line $\frac{x}{6}+\frac{y}{8}=2\frac{1}{2}$. [4]

(b) Determine the equation of the normal at the point $(2,-2)$ to the curve whose equation is $x^2+y^2+3xy+4=0$. [5]

A2. Solve the following equations for $0° \leq \theta \leq 360°$.

(a) $2sin\theta = sec\theta$ [3]

(b) $2cos^2\theta - 7cos\theta + 3 = 0$ [4]

A3. (a) Show that $\frac{1+cos\theta}{1-cos\theta} = cot^2(\frac{\theta}{2})$. [4]

(b) Sketch on the same axis the graph of $y = Inx$ and $y = In(x-1)$ [3]

A4. Evaluate

(a) $\frac{d(x^2sin^{-1}2x)}{dx}$. [4]

(b) $\int \frac{5t^4+4t^2-t+10}{5t^5}dt$. [4]

(c) $\int_1^2 (2x-3)^4 dx$. [4]

A5. The positive quantities x and y are related by the differential equation $\frac{dy}{dx} = (\frac{y}{x})^2$. Find the general solution of this differential equation, expressing y in terms of x. [5]

SECTION B (60 marks)

Candidates may attempt TWO questions being careful to number them B6 to B8.

B6. (a) Given $A = \frac{13}{5}$ and $\cot B = \frac{-8}{15}$, where both A and B are obtuse, find the exact value of $\sin(A - B)$. [7]

(b) The straight line $y = 20 - 3x$ meets the circle $x^2 + y^2 - 2x - 14y = 0$ at the points A and B. Calculate the exact length of the chord AB. [6]

(c) (i) Prove the identity $tan(\theta + 45°) + \tan(\theta - 45°) \equiv 2\tan 2\theta$. [5]

(ii) Hence, find the acute angle θ such that $tan(\theta + 45°) + tan(\theta - 45°) = 2$. [3]

(d) Find the area enclosed by the curve $y = cotx$, the $x - axis$ and the lines $x = \frac{\pi}{3}$ and $x = \frac{\pi}{2}$, giving your answer to three significant figures. [4]

(e) Solve the differential equation $\frac{dy}{dx} = (1 - x)(1 + y)$. [5]

B7. (a) Use the substitution $u = 2x + 3$ to find $\int \frac{x}{(2x+3)^3} dx$. [5]

(b) Use integration by parts to find the exact value of $\int_1^e (Inx)^2 dx$. [5]

(c) Find

(i) $\int cos^2 3x dx$. [3]

(ii) $\int tan^2 2x dx$. [3]

(d) Express $3cos\theta - 5sin\theta$ in the form $Rcos(\theta + \alpha)$, where $\mathbf{R} > 0$ and $0 < \alpha < 90°$. [5]

(e) Hence or otherwise, find the general solution of the equation $3cos\theta - 5sin\theta = 2$, giving your answer correct to the nearest 0.1°. [5]

(f) Solve the equation $tan3x = 11tanx$, giving all solutions such that $0 \leq x \leq \Pi$. [4]

B8. (a) From the graph of $f(x) = x^2$, sketch on separate diagrams:

(i) $f(x - a)$ [2]

(ii) $f(x) + a$ [2]

(b) Prove that $cos3\theta = 4cos^3\theta - 3cos\theta$. [6]

(c) Find the centre and radius of the circle $2x^2 + 2y^2 - 8x - 7y = 0$ using completing the square method. [5]

(d) Find the equation of the tangent at the point $(0, 2)$ to the circle $x^2 + y^2 - 4x + 2y - 8 = 0$. [6]

(e) Find the equation of the normal to the curve $y = 2x^2 - 7x + 2$, at the point $(1, -3)$. [5]

(f) Find the MacLaurin expansion of e^{2x} in ascending powers of x up to the term in x^3. [4]

END OF QUESTION PAPER