BINDURA UNIVERSITY OF SCIENCE EDUCATION

MT107: CALCULUS

Time : 3 hours

Answer ALL questions in Section A and at most TWO questions in section B.

SECTION A (40 marks)

Candidates may attempt ALL questions being careful to number them A1 to A4.

A1. (a) Evaluate: $\int_1^3 dx \int_2^4 (40 - 2xy)dy$ [7]

(b) Solve the following inequalities giving your solution in interval notation.
$\dfrac{m+4}{2} - \dfrac{4m+3}{5} > 2.$ [3]

A2. (a) Find Df and Rf if $f(x) = \dfrac{1}{\sqrt{3-x}}$. [4]

(b) When do we say a sequence S_n is convergent. [2]

(c) Determine whether the sequence $S_n = \dfrac{(-1)^{n+1}(n^2)}{n^2 + 1}$, where $n = 1, 2, 3..$ converges or diverges. [4]

A3. (a) State the conditions that must be satisfied by the Rolle's Theorem. [3]

(b) A box shape X is described by the triple integral:
$X = \int_0^3 \int_0^2 \int_0^1 (x + y + z)dzdydx$. Evaluate X. [7]

A4. (a) Find the area of the region bounded by $f(x) = 4 - 4x^2$ and $g(x) = 1 - x^2$. [5]

(b) Verify the Mean Value Theorem for $f(x) = x^2$ where $a = 0$ and $b = 1$. [5]

SECTION B (60 marks)

Candidates may attempt TWO questions being careful to number them B5 to B7.

B5. (a) Give a detailed sketch of the graph of $y = \dfrac{x^3}{3x-2}$. [12]

(b) Let $f(x) = x^2 \sin(\dfrac{1}{x}), x \neq 0$.

    (i) Does $f(x)$ have a derivative at $x = 0$? Justify your answer. [4]

    (ii) Is $f(x)$ differentiable at $x = 0$, justify your answer. [4]

(c) Find the dimensions of an isosceles triangle of largest area that can be inscribed in a circle of radius a units. [10]

B6. (a) State Stoke's Theorem. [3]

(b) Let C be the oriented triangle lying on the plane $2x + 2y + z = 6$. Use Stoke's Theorem to evaluate $\int_C F.ds$. [7]

(c) Use the iterated integral to find the area of the region bounded by the graphs of $f(x) = \sin(x)$ and $g(x) = \cos(x)$ between $x = \frac{\pi}{4}$ and $x = \frac{5\pi}{4}$. [10]

(d) Give five properties of double integrals. [10]

B7. (a) Obtain a reduction formula for the indefinite integral $I_n = \int x^n \cos(x)dx$ and hence determine I_3. [10]

(b) Find the indefinite integral of $\int \dfrac{x^3 + 2}{x^3 - x}dx$. [7]

(c) State the $\epsilon - N$ definition of the limit of a sequence a_n. [2]

(d) Hence show that a sequence whose n^{th} term given by $a_n = (3 - \frac{1}{7n^2})$ converges to 3. [5]

(e) Show that the sequence $U_n = \dfrac{2n - 7}{3n + 2}$ is monotonic increasing. [6]

<center>**END OF QUESTION PAPER**</center>