JUN 2025

BINDURA UNIVERSITY OF SCIENCE EDUCATION

STATISTICS AND MATHEMATICS

SFM 412: ACTUARIAL MATHEMATICS

Time: 3 hours

Candidates may attempt any **THREE** questions. Each question should start on a **fresh** page.

1. (a) Prove that

$$d < d^{(2)} < d^{(3)} < ... < \delta < ...i^{(3)} < i^{(2)} < i$$ [5]

(b) The force of interest is given by:

$$\delta(t) = \begin{cases} 0.09 + 0.0006t^2 & 0 \leq t < 9 \\ 0.1836 & 9 \leq t < 15 \\ 0.1086 - 0.005t & 15 \leq t < 20 \\ 0.1086 & 20 \leq t \end{cases}$$

where t is measured in years.
Calculate the accumulated value at time 17 of a continuous payment stream with rate $\rho(t) = e^{0.08+0.0002t^3}$, payable for seven years starting at time 2. [7]

(c) An insurance company has liabilities of $100,000 due in four years' time and $200,000 due in 14 years' time. The company owns assets consisting of five coupon bearing bonds of $10,000 nominal each and one zero coupon bond paying $X in n years' time. The coupon bearing bonds pay 4% coupons annually in arrear for five years and are redeemable at par.
i. The current effective interest rate is 6% per annum and an attempt has been made to immunise the portfolio against small movements in interest rates. Determine X and n. [6]
ii. Without doing further calculations, state whether Redington's third condition for immunisation holds, giving a reason for your answer. [2]

2. Let i be the annual effective interest rate and n and p be any natural numbers.

(a) Define algebraically in a summation format, $\ddot{a}^{(p)}_{\overline{n}|i}$ [1]

(b) Prove that $\ddot{a}^{(p)}_{\overline{m+n}|i} = \ddot{a}^{(p)}_{\overline{m}|i} + \nu^m \ddot{a}^{(p)}_{\overline{n}|i}$ [3]

(c) Calculate ${}_{4.25}q_{87.25}$ assuming $PMA92C20$ mortality and that there is a uniform distribution of deaths between integer ages. You should show your working. [3]

(d) Below is an extract from an African country:

x	d_x
64	2569
65	2896
66	3436
67	3977

The number of lives alive at age 65 is 98529.
Calculate ${}_{0.8}P_{65.5}$ assuming:

i. Uniform distribution of deaths (UDD). [5]

ii. Constant force of mortality (CFM). [4]

(e) Prove that

$$ {}_{t-s}q_{x+s} = \frac{(t-s)q_x}{1 - sq_x} \quad [4]$$

3. (a) A mortality table gives the value of $A_{50} = 0.3$ at an interest rate of 5% p.a. effective. Calculate, using appropriate approximations where necessary and showing all working, the values for:

i.
$$\bar{A}_{50} \quad [1]$$

ii.
$$a_{50} \quad [1]$$

iii.
$$\bar{a}_{50} \quad [1]$$

iv.
$$\ddot{a}_{50}^{(4)} \quad [2]$$

using the same basis.

(b) A loan is repaid, over a term of 20 years, by monthly instalments paid in arrears. During the first year, the annual rate of payment is \$5,000. This rate then reduces by \$300 at the start of each subsequent year of the term, up to and including the start of the 10th year, after which the rate remains unchanged.

The instalments were calculated using a nominal rate of interest of 9% p.a. convertible monthly for the first 10 years and 10.5% p.a. convertible monthly for the remainder of the term.

i. Calculate, using appropriate annuity factors and showing all working, the amount of the loan. [9]

ii. Calculate, using appropriate annuity factors and showing all working, the capital and interest components of the 6th monthly instalment. [6]

4. (a) Prove that

i.

$$i + i = \left(1 + \frac{i^{(m)}}{m}\right)^m$$ [5]

ii.

$$1 - d = \left(1 - \frac{d^{(m)}}{m}\right)^m$$ [5]

(b) Describe in words ${}_{a|b}q_{[x]+1}$. [2]

(c) Calculate, showing all working, ${}_{5|3}q_{[38]+1}$.

Basis:
Mortality AM92 (select) [2]

(d) Calculate $a^{(4)}_{82\frac{1}{4}}$, showing all working.

Basis:

Mortality	PMA92C20
Interest	4% per annum effective

[6]

5. (a) Calculate, using standard approximations where necessary:
   i. $\bar{A}^{\,1}_{46:\overline{25}|}$
   ii. ${}_{10|4}q_{[36]}$

   Basis:

Mortality	AM92
Interest	4% per annum effective

   [5]

   (b) Derive from first principles the following:
   i. The variance of a whole life assurance contract. [7]
   ii. The variance of a guaranteed annuity contract. [8]

END OF EXAM