SFM412

BINDURA UNIVERSITY OF SCIENCE EDUCATION

HBsc Statistics and Financial Mathematics

SFM412: Actuarial Mathematics

Time: 3 hours

Candidates may attempt **ALL** questions in Section A and at most **TWO** questions in Section B. Each question should start on a fresh page.

SECTION A (40 marks)
Candidates may attempt ALL questions being careful to number them 1 to 3.

1. The annual force of interest, $\delta(t)$, is a function of time t, where t is measured in years) and is given by:

$$\delta(t) = 0.002t^3$$

   (a) Calculate the equivalent constant annual effective interest rate applicable between time two and four. [3]

   (b) Calculate the equivalent constant force of interest applicable between time two and four. [4]

   (c) Calculate the combined present value of

   . an annuity paying $R1,000$ per annum in advance for a period of three years, with the first payment made at time zero and

   . a lumpsum payment of $R3,000$ made at time four. [6]

2. The notation $a_{\overline{n}|}^{(p)}$ denotes the present value at time 0 of a level annuity, payable p-thly in arrears at a rate of 1 per unit time, over the time interval $[0, n]$. For this annuity, the payments are made at time $\frac{1}{p}, \frac{2}{p}, \frac{3}{p},...,n$ and the amount of each payment is $\frac{1}{p}$, with n a positive integer.

   (a) Derive, from first principles, $a_{\overline{n}|}^{(p)} = \dfrac{1 - \nu^n}{i^{(p)}}$. Define all notation not defined. [4]

(b) An investor wants to invest a lump sum of X into a fund today and withdraw an amount of $R50,000$ every four years over a period of 60 years. After 60 years the fund should be completely drawn down. The first withdrawal will take place four years from today. The fund will pay compound interest of 7.5% per annum effective. Calculte X using the formula in (a) above. [6]

3. Evaluate the mean and variance of the present value random variable of a deferred whole life assurance contract. [7]

4. (a) State the conditions for Reddington immunisation. [3]

   (b) Derive the mean and variance for the present value random variable of a temporary annuity due. [7]

SECTION B (60 marks)

Candidates may attempt **TWO** questions being careful to number 5 to 7

5. (a) A general insurance company has set up a generalised cashflow model for the claims payments it will pay arising from its portfolio of car insurance policies.

    Describe the cashflows with reference to the certainty and uncertainty of size and timings of payments from the insurance company's point of view. [3]

   (b) Calculate $a^{(4)}_{82\frac{1}{4}}$, showing all your working.

    Basis:

    Mortality PMA92C20

    Interest 4% per annum effective [7]

   (c) Write down the formula for the variance of the present value of an n-year temporary annuity of $1 p.a. payable at the end of each year, issued to a life aged x exact. [3]

   (d) Write down the formula for the variance of the present value of an 20-year temporary annuity of $5,000$ p.a. payable at the end of each year, issued to a life aged 44 exact.

    Basis:

    Mortality AM92 Ultimate

    Interest 4% per annum effective

    [4]

   (e) An organisation is required to pay $5 million in 18 years' time. It holds a cash fund equal to the present value of the liabilities and wishes to use the whole fund to purchase a combination of 9-year and 20-year zero-coupon bonds in order to immunise against small changes in the interest rate. The current interest rate is 8% per annum effective.

    (i) Calculate, showing all working, the proportion of the cash fund that should be invested in each of the two zero-coupon bonds. [8]
    The organisation invests in the two bonds in accordance with the proportions calculated in (i)

(ii) Calculate, showing all working, the proportion of the cash fund that should be invested in each of the two zero-coupon bonds. [5]

6. A government issues a fixed interest bond paying coupons at a rate of 9% per annum, payable half-yearly in arrears. The bond is to be redeemed at $110 per $100 nominal on any coupon payment date from 10 to 15 years after issue. The date of redemption is at the discretion of the government. Investor A is subject to income tax at 25% and capital gains tax at 30% and wishes to achieve a net redemption yield of at least 6% per annum effective.

(a) Calculate, showing all working, the maximum price per $100 nominal that Investor A should offer for this bond on issue. [7]

Investor A purchases the bond at the price determined in part (a). Three years after issue, immediately after a coupon payment has been made, Investor A decides to sell the bond to Investor B. Investor B is subject to income tax at 10% and capital gains tax at 35%, and wishes to obtain a net redemption yield of at least 8% per annum effective.

(b) Calculate, showing all working, the maximum price per $100 nominal that Investor A can expect to receive from Investor B. [10]

The bond is sold to Investor B at the price determined in part (b).

(c) Calculate, using linear interpolation, the net effective annual redemption yield that will be obtained by Investor A. You must show all your working. [5]

(d) A student wishes to investigate whether the pass rate of first year actuarial science students at universities is correlated to a mark obtained in a mathematical entrance exam.

Describe the key steps to follow in the data analysis process, for this investigation, and give an example, applicable in this scenario, for each of the steps. [8]

7. (a) What do you understand by the following terms with regards to spot rates:
   (i) market segmentation theory. [2]
   (ii) liquidity preference theory. [2]
   (iii) expectations theory. [2]

(b) A pension has a liability of $400,000 due in ten years time.

The pension fund has exactly enough funds to cover the liability based on an effective rate of interest of 8% per annum. This is also the interest rate at which current market prices are calculated and the rate earned on cash.

The pension wishes to hold 10% of its funds in cash, and to invest the balance in the following securities:

(i) a zero-coupon bond redeemable at par in 12 years time

(ii) a fixed coupon bond paying half yearly coupons of 8% per annum, in arrear, redeemable at 110% in 16 years time.

Determine the amount to invest in the zero-coupon bond and the fixed-coupon bond, respectively, for the pension fund's portfolio of assets and liabilities to satisfy Redington's first two conditions for immunisation. [10]

Explain, without further calculation, whether the pension fund would be immunised against small changes in the interest rate if the quantities of stock in i. are purchased. [4]

(c) Derive from first principles the variance of the present value random variable of a deferred whole assurance life contract. [10]

END OF EXAM