BINDURA UNIVERSITY OF SCIENCE EDUCATION

DME001: INTRODUCTORY MATHEMATICS

Time : 3 hours JAN 2025

Candidates may attempt ALL questions in Section A and at most TWO questions in Section B. Each question should start on a fresh page.

SECTION A (40 marks)

Candidates may attempt ALL questions being careful to number them A1 to A6.

A1. Simplify the following:

(a) $(6-\sqrt{3})+(5+4\sqrt{3})$ [2]

(b) $(\sqrt{2}+\sqrt{5})(\sqrt{2}-\sqrt{5})$ [3]

(c) $\dfrac{3+\sqrt{2}}{\sqrt{2}-5}$ [3]

A2. (a) Find the remainder when $3x^3-x^2-5x+2$ is divided by $3x+2$. [2]

(b) Given x^2+2x-3 is a factor of $f(x)$, where $f(x)\equiv x^4+6x^3+2ax^2+bx-3a$. Find the value of a and of b. [4]

A3. (a) Evaluate the following indicial expression, giving the final answers as an exact simplified fraction: $(25^{\frac{1}{2}}+81^{\frac{1}{4}})^{\frac{2}{3}}$ [3]

(b) Solve the following logarithmic equation for x, $\log_a(x^2-10)-\log_a x=2\log_a 3$. [4]

A4. (a) Find the set of values of x , that satisfy the following inequality.
$\dfrac{x}{2x-1}\geq 1$ [4]

(b) Express $\dfrac{-12x+5x^2-1}{(x+3)(x-1)^2}$ into partial fractions. [5]

A5. Given that $z_1=13+6i$ and $z_2=8-2i$, find:

(a) $z_1 - z_2$ [1]

(b) $z_1 z_2$ [2]

(c) $\dfrac{z_1}{z_2}$ [2]

A6. A curve is given by $y^3 + y^2 + y = x^2 - 2x$. Show that at the origin, $\dfrac{dy}{dx} = -2$ and $\dfrac{d^2y}{dx^2} = -6$, and give Maclaurin's series for y as far as the term in x^2. [5]

SECTION B (60 marks)

Candidates may attempt TWO questions being careful to number them B7 to B9.

B7. (a) Find $\dfrac{dy}{dx}$ in each of the following:

(i) $y = 3x^4 + 6x - 504.$ [2]

(ii) $y = 4Sin^2 3x.$ [4]

(iii) $y = In(2x^2 + 1).$ [4]

(b) A curve is defined by the parametric equations:
$x = 120t - 4t^2$ and $y = 60t - 6t^2$.
Find the value of $\dfrac{dy}{dx}$ at each of the points where the curve crosses the $x - axis$. [9]

(c) Find the coordinate of the turning point whose equation is $y = 2x^2 + 8x - 9$ and determine the nature. [11]

B8. (a) Express the equation $5sin2x = 4cos2x$ in the form $tan2x = k$ where k is a constant. [2]

(b) Integrate the following expressions with respect to x:

(i) $3x^3 - 4x^{\frac{1}{2}} + 24x - 2$ [2]

(ii) $\frac{x^3+4}{x^2}$ [3]

(iii) $\frac{2}{2x+3}$ from 0 up to 3. [3]

(iv) xe^{4x} from 0 up to 1. [8]

(c) A curve has an equation $y = (4-x^2)^{-\frac{1}{2}}$ for $-1 \leq x \leq 1$. The region R is enclosed by $y = (4 - x^2)^{-\frac{1}{2}}$, the x-axis and the line $x = -1$ and $x = 1$. Find the exact value of the area R. [5]

(d) The sum to infinity of a geometric series is three times the first term of the series. The first term of the series is a.

(i) Show that the common ratio of the geometric series is $\frac{3}{2}$ [2]

(ii) The third term of the geometric series is 81.

(a) Find the sixth term of the series. [2]

(b) Find the value of a as a fraction. [3]

B9. (a) Show that $\sin(75^0) = \frac{1+\sqrt{3}}{2\sqrt{2}}$ [5]

(b) Show that $\tan(A+B) = \frac{\tan(A)+\tan(B)}{1-\tan(A)\tan(B)}$ [9]

(c) Use Taylor series to expand $sin(x + \frac{\Pi}{3})$ in ascending powers of x as far as the power of the term in x^4. [10]

(d) Using the binomial expansion, or otherwise, express $(1+2x)^4$ in the form $1+ax+bx^2+32x^3+16x^4$ where a and b are integers. [6]

END OF QUESTION PAPER