BINDURA UNIVERSITY OF SCIENCE EDUCATION
BACHELOR OF SCIENCE EDUCATION DEGREE
PHYSICS PART 2
PH202 : QUANTUM PHYSICS 1
DURATION: 3 HOURS

JAN 2025

INSTRUCTIONS:

Answer ALL parts of Section A and any THREE questions from Section B.

Section A carries 40 marks and Section B carries 60 marks.

Electron charge,	$e = 1.60 \times 10^{-19}\,C$
Planck's constant,	$h = 6.63 \times 10^{-34}\,Js$
Mass of an electron,	$m_e = 9.11\,x\,10^{-31}\,kg$
Acceleration due to gravity,	$g = 9.81\,ms^{-2}$
Permittivity of free space,	$\varepsilon_0 = 8.85 \times 10^{-12}\,Fm^{-1}$
Speed of light,	$c = 3 \times 10^{8}\,ms^{-1}$

SECTION A

(a) List the five postulates of quantum mechanics [5]

(b) Derive the Compton shift equation $\Delta\lambda = \lambda' - \lambda = \frac{h}{m_e c}(1 - \cos\theta)$. [4]

(c) Find the total energy of the first excited state of the harmonic oscillator [4]

(d) An electron is bound to a region of space by a spring like force with an effective spring constant of $k = 95.7eV/nm^2$.

(i) What is its ground-state energy? [3]

(ii) How much energy must be absorbed for the electron to jump from the ground state to the second excited state? [3]

(e) Consider a particle whose normalized wave function is

$$\psi(x) = 2\alpha\sqrt{\alpha}\, xe^{-\alpha x} \qquad x > 0$$
$$= 0 \qquad x < 0$$

Calculate (i) $\langle x \rangle$ [3]

(ii) $\langle x^2 \rangle$ [2]

(f) (i) An electron is confined to a region of the size of an atom $(0.1nm)$.

What is the minimum uncertainty of the momentum of the electron? [3]

(ii) What is the kinetic energy of an electron with a momentum equal to this

this uncertainty. [2]

(g) State *Ehrenfest*'s principle [1]

(h) Compute the wavelength of H_β spectral lines (i.e. the second line of the

Balmer Series predicted by Bohr's model of atomic structure). The H_β line

is emitted in the transition from n_i = 4 to n_f =2. [4]

(i) Using sketch diagrams, describe Ultraviolet Catastrophe [2]

(j) What is an Eigenvalue equation? [2]

SECTION B

2(a) Differentiate between Bound state and unbound state as explained

in quantum physics [4]

(b) A particle of mass m is confined to a one-dimensional line of length L.

From the arguments based on the wave interpretation of matter

(i) Show that the energy of the particle can have only discrete values

(ii) Determine the values referred in (i) [6]

(c) The wave function ψ of a particle is given by $\psi = Ae^{-kx}$

for $0 < x < \infty$

$\psi = 0$ for $-\infty < x < 0$

(i) Find A in terms of k

(ii) Determine the probability of the particle lying in the region $\frac{2}{k} < x < \frac{3}{k}$ [10]

3(a) A particle in the infinite square well has the initial wave function

$$\Psi(x, 0) = \begin{cases} Ax, & 0 \le x \le a/2, \\ A(a - x), & a/2 \le x \le a. \end{cases}$$

(i) Sketch (x, t) , and determine the constant *A* [3]

(ii) Find $\Psi(x, t)$ [4]

(iii) What is the probability that a measurement of the energy would yield the Value ? [3]

(b) A Solution to the Schrödinger Equation Show that for a free particle of mass *m* moving in one dimension, the function $\psi(x) = Asinkx + Bcoskx$ is a solution to the time-independent Schrodinger equation for any values of the constants *A* and *B* [5]

(c) Given a one-dimensional box of width 1mm.What value of n corresponds to a state of energy 0.01eV [5]

4 (a) Write the normalized wave equation for a particle trapped in a box [2]

(b) Find the probability that a particle trapped in a box L wide can be found between $0.45L \; and \; 0.55L$ for the following states

(i) ground state

(ii) first excited state [8]

(c) (i) (f) State and explain THREE quantum mechanics modifications made to the classical picture of Simple Harmonic oscillations [4]

(ii) Using clearly labeled sketch diagrams compare the energy levels of infinite potential well, Harmonic oscillator and hydrogen atom [6]

5(a)Write the momentum operator in one dimension [2]

(ii) Show that the Schrödinger's time-independent equation is just the eigenvalue equation for energy [3]

(b)(i) Show that the wave function $\psi_{(x)} = Ae^{ikx}$ represents a state for which the momentum of the particle has the value $p = \hbar k$ [4]

(ii) Find the kinetic energy of the particle in this state [3]

(c) In a region of space a particle with mass m and zero energy has a time-independent wave function

$$\psi_{(x)} = Axe^{-x^2/L^2}$$

Where A and L are constants

Determine the potential energy $U(x)$ of the particle [8]

6 (a) The Heisenberg uncertainty is often viewed as a limitation to measurements, but in combination with energy minimization it also has significant power in predicting the magnitude of quantum mechanical effects. Assume that you have a particle of mass *m* in a harmonic potential

$$V(x) = \frac{1}{2}m\omega^2x^2.$$

(i) Argue in one or two sentences why a particle localized in a very narrow region around $x = 0$ would have a large total energy. [4]

(ii) By minimizing the total energy subject to the Heisenberg uncertainty relation between position spread Δx and momentum spread Δp the particle, calculate the size Δx of the ground state in the harmonic potential [6]

b Show that in the nth eigenstate of the harmonic oscillator, the average kinetic energy (T) is equal to the average potential energy *(V)* (the virial theorem) . That is,

$$\langle V\rangle = \frac{K}{2}\langle x^2\rangle = \langle T\rangle = \frac{1}{2m}\langle p^2\rangle = \frac{1}{2}\langle E\rangle = \frac{\hbar\omega_0}{2}\left(n+\frac{1}{2}\right)$$ [10]

END OF EXAM PAPER