BINDURA UNIVERSITY OF SCIENCE EDUCATION

FACULTY OF SCIENCE AND ENGINEERING

DEPARTMENT OF STATISTICS AND MATHEMATICS

EEE2201/MTE1201: ENGINEERING MATHEMATICS 2
ENGINEERING MATHEMATICS 2B

DURATION: 3 HOURS **TOTAL MARKS: 100** JUN 2025

INSTRUCTIONS TO CANDIDATES

Answer ALL questions in Section A and any TWO questions from Section B
The number of marks is indicated in brackets at the end of each question
Each question should start on a fresh page

SECTION A [40 MARKS]

A1. Define the following terms

a) Ordinary differential equation [2]
b) Partial differential equation [2]
c) Invertible Matrix [2]

A2. Find the rank of the matrix $A = \begin{bmatrix} 2 & 3 & 7 \\ 3 & -2 & 4 \\ 1 & -3 & -1 \end{bmatrix}$ [6]

A3. Solve the following exact differential equation
$\left[y\left(1+\frac{1}{x}\right)+cosy\right]dx+[xlogx-xsiny]dy=0$ [4]

A4. Form the partial differential equation by eliminating the arbitrary constants a and b from
$z=(x+a)(y+b)$ [4]

A5. Find the general solution of $(D^2-3D+4)y=0$ [6]

A6. If a radioactive Carbon-14 has a half-life of 5750 years, what will remain of one gram after 3000 years? [6]

A7. Find the O.T of the family of circles $x^2+y^2+2gx+c=0$, where g is the parameter [8]

SECTION B [60 MARKS]

B8.

a) If $U = \log(x^3 + y^3 + z^3$, prove that $\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}\right)^2 U = \frac{-9}{(x+y+z)^2}$ by the use of total derivatives [12]

b) Use the Jacobian matrix to answer the following,

i. Show that the functions $u = x + y + z, v = x^2 + y^2 + z^2 - 2xy - 2yz - 2xz$ and $w = x^3 + y^3 + z^3 - 3xyz$ are functionally related. [11]

ii. If $U = \frac{yz}{x}, v = \frac{xz}{y}, w = \frac{xy}{z}$ find $\frac{\partial(u,v,w)}{\partial(x,y,z)}$ [7]

B9.

(a) Find whether the following equations are consistent, if so solve them.
$x + y + 2z = 4, 2x - y + 3z = 9, 3x - y - z = 2$ [12]

(b) Find the eigen values and eigen vectors of the matrix A and its inverse, where

$$A = \begin{bmatrix} 1 & 3 & 4 \\ 0 & 2 & 5 \\ 0 & 0 & 3 \end{bmatrix}$$ [18]

B10.

(a) An object whose temperature is 75^0C cools in an atmosphere of constant temperature C, at the rate of k$\boldsymbol{\theta}$, being the excess temperature of the body over that of the temperature. If after 10min, the temperature of the object falls to 65^0C, find its temperature after 20 min. Also find the time required to cool down to 55^0C. Take one minute as unit of time. [12]

(b) A bacterial culture, growing exponentially, increases from 100 to 400 gms in 10 hrs. How much was present after 3 hrs, from the initial instant? [8]

(c) The equation $\frac{d^2i}{dt^2} + \frac{R}{L}\frac{di}{dt} + \frac{1}{LC}i = 0$ represents a current i flowing in an electrical circuit containing resistance R, inductance L and capacitance C connected in series. If $R = 200$ ohms, $L = 0.20$ henry and $C = 20 \times 10^{-6}$ farads. Solve the equation for i given the boundary conditions that when $t = 0, i = 0$ and $\frac{di}{dt} = 100$ [10]

END OF PAPER