BINDURA UNIVERSITY OF SCIENCE EDUCATION BTEC123

BACHELOR OF STATISTICS AND FINANCIAL MATHEMATICS

MATHEMATICS FOR TECHNOLOGISTS

Time : 3 hours JUN 2024

Candidates may attempt ALL questions in Section A and at most TWO questions in Section B. Each question should start on a fresh page.

SECTION A (40 marks)

Candidates may attempt ALL questions being careful to number them A1 to A4.

A1. State and explain the Gauss elimination steps for solving a system of linear equations. [8]

A2. Define the following terms

(a) Trace of a matrix. [2]

(b) Rank of a matrix. [2]

(c) Singular matrix. [2]

A3. Use the Gauss- Jordan method to solve the following system of linear equations [8]

$$\begin{aligned} x + y - z &= 6 \\ 3x - 2y + z &= -5 \\ x + 3y - 2z &= 14 \end{aligned}$$

A4. (a) Given $f(x, y) = \cos xy - e^{\sin x}$, evaluate

(i) f_x [2]

(ii) f_{xx} [3]

(iii) f_{xy} [3]

(b) Given that $\mathbf{a} = 2\mathbf{i} - 3\mathbf{j} + 14\mathbf{k}$ and $\mathbf{b} = 5\mathbf{i} - 8\mathbf{j} + 11\mathbf{k}$, evaluate:

(i) $\mathbf{a}.\mathbf{b}$ [3]

(ii) a × b [3]

(iii) 2a − 6b [2]

(iv) 8a − 12b [2]

SECTION B (60 marks)

Candidates may attempt TWO questions being careful to number them B5 to B7.

B5. (a) Find the solution to the linear system of equations,

$$2x + y + z = 10$$
$$x + 2y + 3z = 1$$
$$-x - y - z = 2$$

using Cramer's Rule [8]

(b) Consider the vectors $\mathbf{u} = (1, 2, -1)$ and $\mathbf{v} = (6, 4, 2)$ in $\mathbb{R}^3$.Show that $\mathbf{w} = (9, 2, 7)$ is a linear combination of $\mathbf{u}$ and $\mathbf{v}$ and that $\mathbf{w}' = (4, -1, 8)$ is not a linear combination of $\mathbf{u}$ and $\mathbf{v}$ [6]

(c) Prove that if $\mathbf{u}$ and $\mathbf{v}$ are vectors in $\mathbb{R}^n$ with the Euclidean inner product, then

$$\mathbf{u}.\mathbf{v} = \frac{1}{4}||\mathbf{u} + \mathbf{v}||^2 - \frac{1}{4}||\mathbf{u} - \mathbf{v}||^2$$

[6]

(d) If A and B are matrices of the same order, prove that,

(i) $(AB)^T = B^T A^T$ [3]

(ii) $(AB)^{-1} = B^{-1}A^{-1}$ [3]

(e) Define the following terms:

(i) Positive definite matrix [2]

(ii) Indefinite matrix [2]

B6. (a) Evaluate the following double integrals:

(i) $\iint_D (x + 2y)dydx$ where D is the region bounded by the parabolas $y = 2x^2$ and $y = 1 + x^2$ [5]

(ii) $\iint_R ysin(xy)dydx$ where R is the region $R = [1, 2] \times [0, \pi]$ [5]

(b) Evaluate the adjoint and hence the inverse of the matrix: $\begin{pmatrix} 1 & 0 & -1 \\ 2 & 1 & -1 \\ 1 & 2 & 5 \end{pmatrix}$ [7]

(c) Let $A=\begin{pmatrix} 5 & 0 & -7 \\ 14 & 1 & -1 \\ 1 & 4 & 3 \end{pmatrix}$. Evaluate:

(i) A^TA [4]

(ii) A^3 [4]

(iii) A^2 [3]

(iv) A^5 [2]

B7. (a) Let $M=\begin{pmatrix} 2 & 1 & 1 \\ 2 & 0 & -1 \\ 0 & 3 & 1 \end{pmatrix}$. Evaluate

(i) The eigenvalues [7]

(ii) The eigenvectors [8]

(iii) Normalised eigenvectors [3]

(b) Write down the R-code for determining quantities in part (a) above. [12]

END OF QUESTION PAPER