BINDURA UNIVERSITY OF SCIENCE EDUCATION SFM222

HBSc. IN STATISTICS & FINANCIAL MATHEMATICS

RISK THEORY

JUN 2025

Time : 3 hours

Candidates may attempt ALL questions in Section A and at most TWO questions in Section B. Each question should start on a fresh page.

SECTION A (40 marks)

Candidates may attempt ALL questions being careful to number them A1 to A5.

A1. Define the following terms;

(a) ruin time, [2]

(b) compound model, [2]

(c) the principal of zero utility, [2]

(d) reinsurance, [2]

(e) insurance system. [2]

A2. Distinguish between the following concepts;

(a) risk averse and security loading, [2]

(b) individual and collective risk models, [2]

(c) insurance system and gambling, and [2]

(d) utility theory and expected value theory. [2]

A3. Let $X_1, X_2, ..., X_N$ be independent and identically distributed random variables, where the $X_i's$ denote the amount of the i^{th} claim and let $S = X_1 + X_2 + ... + X_N$ with random variables $N, X_1, X_2, ..., X_N$ that are mutually independent. Let $P(x)$ denote the common distribution function of the $X_i's$, and let X be the random variable with this distribution function such that $p_k = E[X^k]$ denoting the k^{th} moment about the origin. Derive the expressions for;

(a) $E[S]$, [3]

(b) $Var[S]$, [4]

(c) $M_S(t)$. [5]

A4. A Roman impluvium is a basin built into a Roman house, placed under an opening in the roof, meant to catch rainwater for the household to use. Suppose that, during a particular storm, the impluvium catches, on average, 20 drops of water per second, and each drop of water caught by the impluvium follows the exponential distribution. Find the probability that the first drop of water caught by the implvium is within the first tenth of a second (.1 second. [5]

A5. Let N be a discrete random variable that gives the probability of n events occurring in s seconds. The occurrence of these events follows the exponential distribution. Prove that

$$P(N(s) = n) = \frac{e^{-\lambda s}(\lambda s)^n}{n!}$$

[8]

SECTION B (60 marks)

Candidates may attempt TWO questions being careful to number them B6 to B8.

B6. (a) The policy holders of an automobile insurance company fall into two classes. The distribution of claim amount, B_k, has a truncated exponential distribution with distribution function.

$$F(x) = \begin{cases} 0 & if\ x < 0 \\ 1 - e^{-\lambda x} & if\ 0 < x < L \\ 1 & if\ x \geq L \end{cases}$$

Class (k)	Number (n_k)	Claim probability (q_k)	λ	L
1	500	0.1	1	2.5
2	2000	0.05	2	5

The probability that total claims exceed the amount collected from policy holders is to be 0.05. Assume that the relative security loading, θ, is to be the same for the two classes. Let $X_1, X_2, ...$ be $i.i.d.$ random variables, where $X_i's$ denote the amount of the i^{th} claim and let $S = X_1 + X_2 + ... + X_n$ with random variables $X_1, X_2, ...$ mutually independent. Let $X = IB$ where X is the claim random variable for the period, B gives the total claim amount incurred during the period, given that at least one claim occurred, and I is the indicator function for the event that at least one claim occurred during the period. That is, $I = 1$ if a claim occurs and $I = 0$ if no claim occurs during the period and $P(I = 1) = q$. For the automobile insurance, find

(i) $\mu = E[B|I = 1]$, [2]

(ii) $E[B^2|I = 1]$, [2]

(iii) $\sigma^2 = Var[B|I = 1]$, [2]
(iv) $E[S] = \sum_{k=1}^{k=n} n_k(\mu_k q_k)$, [5]
(v) $Var(S) = \sum_{k=1}^{k=n} n_k V_k$, where $V_k = \mu_k^2 q_k(1 - q_k) + \sigma_k^2 q_k$, [6]
(vi) the relative security loading, θ. [8]

(b) Consider n people aged x at time $t = 0$. Let $N(t)$ denote the number of deaths that have occurred by time t, and T_i denote the time when the i^{th} death occurs $(i = 1, 2, ..., n)$. We assume independence of the times until death. Specify the process $\{N(t),\ t \geq 0\}$ by the global method. [5]

B7. (a) The probability that a private light motor vehicle will not be damaged in the next period is 0.90. The probability density function of a loss is given by;

$$f(x) = 0.1[0.01e^{-0.01x}], \quad x > 0.$$

The owner of the vehicle has a utility function given by;

$$u(x) = -e^{-0.005x}.$$

Find;

(i) the expected loss, [3]
(ii) the maximum insurance premium the vehicle owner will pay for full insurance, [5]
(iii) the maximum premium that the vehicle owner will pay if he/she is offered an insurance, policy that will pay three quarters of any loss during the next period. [5]

(b) A decision maker has a utility function $u(x) = e^{-\alpha w}$ and is faced with a random loss that has a chi-square distribution with n degrees of freedom. If $0 < \alpha < 0.5$,

(i) obtain an expression for G, the maximum insurance premium the decision maker will pay, [4]
(ii) prove that $G > n = \mu$ [3]

(c) A compound distribution is such that, $P(N = 0) = 0.5$, $P(N = 1) = 0.4$ and $P(N = 2) = 0.1$. Claim amounts are either 1 unit, 2 units or 3 units, with probabilities 0.4, 0.4 and 0.2 respectively. If S is the aggregate claims, then

(i) derive the distribution of S, [5]
(ii) find the expected value of S, [2]
(iii) find the variance of S. [3]

B8. (a) Define the term ruin. [2]

(b) Describe the following methods used to define claim number process;

(i) the global method, [2]

(ii) the infinitesimal method, [2]

(iii) the discrete (waiting time) method. [2]

(c) Let $\psi(u)$ denote the probability of ruin. Prove that for $u \geq 0$,

$$\psi(u) = \frac{e^{-Ru}}{E[e^{-RU(T)}|T < \infty]}.$$

[6]

(d) Suppose that the claim amount follows an exponential distribution with parameter, $\beta > 0$. Find the

(i) adjustment coefficient, [3]

(ii) adjustment coefficient if all claims are of size 1, [3]

(iii) probability of ruin. [4]

(e) Suppose that S has a compound Poisson distribution with $\lambda = 0.8$ and individual claim amounts are 1, 2, or 3 with probabilities 0.25, 0.375 and 0.375 respectively. Find the probability function and distribution function of aggregate claims for $x = 0, 1, ..., 6$. [6]

END OF QUESTION PAPER