BINDURA UNIVERSITY OF SCIENCE EDUCATION

SFM 422: Econometric Modelling JUN 2025

Time: 3 hours

Candidates may attempt ALL questions in Section A and at most two questions in Section B. Each question should start on a fresh page.

Section A (40 marks)

Candidates may attempt ALL questions being careful to number them A1 to A5.

A1. Convert the following into linear models

a) *i)* $y_i = e^{(\beta_1 + \beta_2 x_i + u_i)}$, [2]

ii) $y_i = \frac{1}{1 + e^{(\beta_1 + \beta_2 x_i + u_i)}}$, and [3]

iii) $y_i = \frac{x_i}{\beta_1 + \beta_2 x_i}$. [3]

b) Specify the assumptions of the classical model. [2]

A2. a) Define the type 1 error and p value of a statistical test for H_0 against H_1. [4]

b) Explain the concept of asymptotic size of a test for H_0 against H_1. [3]

A3. a) What is econometrics? [3]

b) Explain the four different types of econometric data. [10]

A4. You estimate the following model:

$y_i = -3.86 + 0.692x_i + 11.8x_i + u_i,$ N=70, R2 = .61

(1.03) (.520) (2.77)

The Durbin-Watson statistic is 1.62. Use a Durbin-Watson test to check for the presence of positive autocorrelation. [5]

A5. If we estimate the following models using OLS, list the problems associated with each model separately.

a) $Y_t = aY_{t-1} + \beta_1 + \beta_2 X_{2t} + \beta_3 X_{3t} + u_t$ and

b) $Y_t = \beta_1 + \beta_2 X_{2t} + \beta_3 X_{3t} + u_t,\ u_t = \rho u_{t-1} + e_t$. [5]

Section B (60 marks)

Candidates may attempt two questions being careful to number them B6 to B8.

B6. Consider a simple regression model,

$y = \beta_0 + \beta_1 x + u$ using cross-sectional data $\{x_i, y_i\}_{i=1}^{n}$.

a) Write down the minimisation problem that defines $\hat{\beta}_0$ and $\hat{\beta}_1$ and its solution. [4]

b) Derive the two first order conditions that characterise $\hat{\beta}_0$ and $\hat{\beta}_1$. [3]

c) Write down the OLS estimator $\hat{\beta}_0$ and $\hat{\beta}_1$ as a function of $\{x_i, y_i\}_{i=1}^{n}$. [4]

d) Explain why we need some variation in x_i for the OLSE to be well defined. [3]

e) Define the residual $\hat{u}_i$. [3]

f) Write down the coefficient of determination, R^2 as a function of the variables defined above. [5]

g) Consider a simple regression model: $\hat{u}_i = \alpha_i x + \varepsilon$, where ε is the error term. Explain why we get zeros for both $\hat{\alpha}_0$ and $\hat{\alpha}_1$. [6]

h) Explain the significance of R^2. [2]

B7. Suppose that we are interested in the relationship between the agricultural sector and total output in developing countries and we are given data on variables *Y* and *X* where *Y* measures per capita GDP in thousands of $US and *X* measures the percentage of the labour force in the agricultural sector. The data is

X	9	10	8	7	10	4	5	5	6	7
Y	6	8	8	7	7	12	9	8	9	10

a) Consider the relationship given by $y_i = \beta_0 + \beta_1 x_i + e_i$ and assume that the Gauss Markov theorem holds. Use this data to calculate b_1, b_2, SSE (the sum of squared errors) and $se(b_2)$. Show your working. [16]

b) Then test the null hypothesis (at the 5% level) that the percentage of workers in the labour force in agriculture does not affect per capita GDP, choosing a suitable alternative for your test. [5]

c) Briefly justify your choice of an alternative hypothesis, and comment on any additional assumptions that you need to make to perform the test. [4]

The results of these estimates are confirmed by Eviews.

Dependent Variable: Y
Method: Least Squares
Sample: 1 10
Included observations: 10

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	**12.28851**	1.534242	8.009498	0.0000
X	**-0.547677**	**0.207824**	-2.635294	0.0299

R-squared	**0.464696**	Mean dependent var	8.400000
Adjusted R-squared	0.397783	S.D. dependent var	1.712698
S.E. of regression	1.329099	Akaike info criterion	3.583736
Sum squared resid	**14.13203**	Schwarz criterion	3.644253
Log likelihood	-15.91868	F-statistic	6.944775
Durbin-Watson stat	1.947873	Prob(F-statistic)	0.029931

d) Test the null hypothesis, at the 5% level of significance, that the percentage of workers in the labour force in agriculture does not affect per capital GDP (with an alternative that it has a negative effect on GDP). [5]

B8. You are given the following situations:

a) Suppose, you are short on time but need to get some econometric work done, so you will have to leave instructions to your assistant. For the model: $y_i = \beta_1 + \beta_2 x_{12} + \beta_3 x_{13} + u_i$, give detailed instructions on how to test the null hypothesis that $\beta_2 - 2\beta_3 = 1$. [10]

b) Suppose you estimate the following model.

$$\hat{Y}_i = 4.89 - 0.87 * X_{2i} - 6.00 * X_{3i} - 11.84 * X_{4i} + u_i.$$

(1.88) (0.32) (2.00) (6.31)

Form the t-statistic to test the hypothesis that the coefficient on X_3 is negative i.e. $H_0: \beta_3 = 0$ versus $H_1: \beta_3 < 0$ at 5% level of significance. [5]

c) i. Find the reduced form for the following model (the endogenous variables are *Y*, *C*, and *NX*, and the exogenous variables are *I*, *G*, *T*, and *P*) [8]

$Y_t = C_t + I_t + G_t + NX_t$,

$C_t = a + b(Y_t - T_t) + u_t$, and

$NX_t = f + gY_t + hP_t + v_t$.

ii. For the model

$Q_t = a_0 + a_1 P_t + a_2 Y_t + a_3 X_t + a_4 Z_t + u_t$,

$P_t = b_0 + b_1 Q_t + b_2 Y_t + b_3 W_t + v_t$, and

$Y_t = c_0 + c_1 P_t + c_2 W_t + w_t$.

Determine whether each equation is under, exactly, or over identified? (Assume that *Q*, *P*, and *Y* are endogenous, and the constant, *X*, *Z*, and *W* are exogenous). [7]

THE END

LIST OF FORMULAE FOR SFM 422

$$S_{xy} = \sum xy - n\bar{x}\bar{y}$$

$$S_{xx} = \sum x^2 - n\bar{x}^2$$

$$S_{yy} = \sum y^2 - n\bar{y}^2$$

$$\hat{\beta}_1 = \frac{S_{xy}}{S_{xx}}$$

$$\hat{\beta}_0 = \bar{y} - \hat{\beta}_1\bar{x}$$

$$t_0 = \frac{\hat{\beta}_1 - \beta_{null}}{\mathrm{SE}(\hat{\beta}_1)}$$

$$R^2 = \frac{S_{xy}{}^2}{S_{xx}S_{yy}} \qquad \text{or} \qquad R^2 = 1 - \mathrm{SSR}/\mathrm{SST}$$

$$F = \frac{(RSS_r - RSS_{ur})}{RSS_{ur}} \ \mathrm{X} \ \frac{df_{ur}}{(df_r - df_{ur})}$$

$$F = \frac{(R^2{}_{ur} - R^2{}_r)/k}{(1 - R^2{}_{ur})/(n - k - 1)}$$

$$SE(\hat{\beta}_1) = \sqrt{\frac{\hat{\sigma}^2{}_e}{\sum x^2 - n\bar{x}^2}}$$

$$\hat{\sigma}^2{}_e = \frac{\sum \hat{e}^2}{n - k}$$