AUG 2024

BINDURA UNIVERSITY OF SCIENCE EDUCATION

Faculty of Science and Engineering

Department of Engineering and Physics

BACHELOR OF SCIENCE EDUCATION HONOURS DEGREE

Physics

PH208

Solid State Physics I

Duration: Three (3) Hours

Answer ALL parts of Section A and any THREE questions from Section B. Section A carries 40 marks and each question of Section B carries 20 marks.

Clearly show ALL working

You may not start to read the questions printed on the subsequent pages until instructed to do so by the Invigilator.

$1\text{Å} = 10^{-10}m = 10\ nm$

Avogadro's number, $N_A = 6.0225 \times 10^{23}\ mol^{-1}$

Boltzmann constant, $k_B = 1.381 \times 10^{-23}\ JK^{-1}$

Permittivity of vacuum, $\varepsilon_0 = 8.8542 \times 10^{-12}\ Fm^{-1}$

Dirac's constant, $\hbar = 1.054 \times 10^{-34}\ kgm^2s^{-1}$

Electron charge, $e = -1.602 \times 10^{-19}\ C$

$$ln(1+x) = \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n} x^n; \quad e^{\pm ix} = cosx \pm isinx; \quad sin^2x = \frac{1}{2}(1 - cos2x)$$

Lennard-Jones potential

$$U_{total}(R) = 2N\epsilon \left[\sum_j \left(\frac{\sigma}{p_{ij}R} \right)^{12} - \sum_j \left(\frac{\sigma}{p_{ij}R} \right)^{6} \right]$$

	$\sum_j p_{ij}^{-12}$	$\sum_j p_{ij}^{-6}$
bcc	9.11418	12.2533
fcc	12.13188	14.45392

Moseley's law

$$\sqrt{\nu} = a(Z-1)$$

Periodic Table of the Elements

Group I	Group II								
		Transition elements							
H 1 1.007 9 $1s$									
Li 3 6.941 $2s^1$	Be 4 9.0122 $2s^2$								
Na 11 22.990 $3s^1$	Mg 12 24.305 $3s^2$								
K 19 39.098 $4s^1$	Ca 20 40.078 $4s^2$	Sc 21 44.956 $3d^14s^2$	Ti 22 47.867 $3d^24s^2$	V 23 50.942 $3d^34s^2$	Cr 24 51.996 $3d^54s^1$	Mn 25 54.938 $3d^54s^2$	Fe 26 55.845 $3d^64s^2$	Co 27 58.933 $3d^74s^2$	
Rb 37 85.468 $5s^1$	Sr 38 87.62 $5s^2$	Y 39 88.906 $4d^15s^2$	Zr 40 91.224 $4d^25s^2$	Nb 41 92.906 $4d^45s^1$	Mo 42 95.94 $4d^55s^1$	Tc 43 (98) $4d^55s^2$	Ru 44 101.07 $4d^75s^1$	Rh 45 102.91 $4d^85s^1$	
Cs 55 132.91 $6s^1$	Ba 56 137.33 $6s^2$	57–71*	Hf 72 178.49 $5d^26s^2$	Ta 73 180.95 $5d^36s^2$	W 74 183.84 $5d^46s^2$	Re 75 186.21 $5d^56s^2$	Os 76 190.23 $5d^66s^2$	Ir 77 192.2 $5d^76s^2$	
Fr 87 (223) $7s^1$	Ra 88 (226) $7s^2$	89–103**	Rf 104 (261) $6d^27s^2$	Db 105 (262) $6d^37s^2$	Sg 106 (266)	Bh 107 (264)	Hs 108 (277)	Mt 109 (268)	

Symbol — Ca 20 — Atomic number
Atomic mass† — 40.078
$4s^2$ — Electron configuration

*Lanthanide series	La 57 138.91 $5d^16s^2$	Ce 58 140.12 $5d^14f^16s^2$	Pr 59 140.91 $4f^36s^2$	Nd 60 144.24 $4f^46s^2$	Pm 61 (145) $4f^56s^2$	Sm 62 150.36 $4f^66s^2$	
**Actinide series	Ac 89 (227) $6d^17s^2$	Th 90 232.04 $6d^27s^2$	Pa 91 231.04 $5f^26d^17s^2$	U 92 238.03 $5f^36d^17s^2$	Np 93 (237) $5f^46d^17s^2$	Pu 94 (244) $5f^67s^2$	

Note: Atomic mass values given are averaged over isotopes in the percentages in which they exist in nature.
† For an unstable element, mass number of the most stable known isotope is given in parentheses.

			Group III	Group IV	Group V	Group VI	Group VII	Group 0
							H 1 1.007 9 $1s^1$	He 2 4.002 6 $1s^2$
			B 5 10.811 $2p^1$	C 6 12.011 $2p^2$	N 7 14.007 $2p^3$	O 8 15.999 $2p^4$	F 9 18.998 $2p^5$	Ne 10 20.180 $2p^6$
			Al 13 26.982 $3p^1$	Si 14 28.086 $3p^2$	P 15 30.974 $3p^3$	S 16 32.066 $3p^4$	Cl 17 35.453 $3p^5$	Ar 18 39.948 $3p^6$
Ni 28 58.693 $3d^8 4s^2$	Cu 29 63.546 $3d^{10} 4s^1$	Zn 30 65.41 $3d^{10} 4s^2$	Ga 31 69.723 $4p^1$	Ge 32 72.64 $4p^2$	As 33 74.922 $4p^3$	Se 34 78.96 $4p^4$	Br 35 79.904 $4p^5$	Kr 36 83.80 $4p^6$
Pd 46 106.42 $4d^{10}$	Ag 47 107.87 $4d^{10} 5s^1$	Cd 48 112.41 $4d^{10} 5s^2$	In 49 114.82 $5p^1$	Sn 50 118.71 $5p^2$	Sb 51 121.76 $5p^3$	Te 52 127.60 $5p^4$	I 53 126.90 $5p^5$	Xe 54 131.29 $5p^6$
Pt 78 195.08 $5d^9 6s^1$	Au 79 196.97 $5d^{10} 6s^1$	Hg 80 200.59 $5d^{10} 6s^2$	Tl 81 204.38 $6p^1$	Pb 82 207.2 $6p^2$	Bi 83 208.98 $6p^3$	Po 84 (209) $6p^4$	At 85 (210) $6p^5$	Rn 86 (222) $6p^6$
Ds 110 (271)	Rg 111 (272)	Cn 112 (285)	113[†] (284)	Fl 114 (289)	115[†] (288)	Lv 116 (293)	117[†] (294)	118[†] (294)

Eu 63 151.96 $4f^7 6s^2$	Gd 64 157.25 $4f^7 5d^1 6s^2$	Tb 65 158.93 $4f^8 5d^1 6s^2$	Dy 66 162.50 $4f^{10} 6s^2$	Ho 67 164.93 $4f^{11} 6s^2$	Er 68 167.26 $4f^{12} 6s^2$	Tm 69 168.93 $4f^{13} 6s^2$	Yb 70 173.04 $4f^{14} 6s^2$	Lu 71 174.97 $4f^{14} 5d^1 6s^2$
Am 95 (243) $5f^7 7s^2$	Cm 96 (247) $5f^7 6d^1 7s^2$	Bk 97 (247) $5f^8 6d^1 7s^2$	Cf 98 (251) $5f^{10} 7s^2$	Es 99 (252) $5f^{11} 7s^2$	Fm 100 (257) $5f^{12} 7s^2$	Md 101 (258) $5f^{13} 7s^2$	No 102 (259) $5f^{14} 7s^2$	Lr 103 (262) $5f^{14} 6d^1 7s^2$

[†] Elements 113, 115, 117, and 118 have not yet been officially named. Only small numbers of atoms of these elements have been observed.
Note: For a description of the atomic data, visit *physics.nist.gov/PhysRefData/Elements/per_text.html*.

SECTION A

1. (a) Comment on the validity of the assertion that *X-ray production is the inverse of the photoelectric effect.*

[2]

(b) When illuminated by X-rays, explain why the X-ray diffraction spectra for ordinary glass (composition SiO_2) exhibit one or two diffuse rings on the recording film whilst glass in the form of quartz (composition SiO_2) exhibits a large number of rings.

[3]

(c) The X-ray spectrum of a cobalt target $(Z - 27)$ contains a strong K_α line of wavelength 0.1785 nm and weak K_α lines having wavelengths of 0.2285 nm and 0.1537 nm due to impurities. Use Moseley's law to identify the impurities. The constant for K-series may be taken as unity.

[10]

(d) Tungsten is a metal with the body centred cubic (bcc) structure. Its density is 19.25 gcm^{-3} and its atomic weight is $183.84 gmol^{-1}$.

(i) Find the nearest neighbour and second nearest neighbour distances in tungsten.

[6]

(ii) How many nearest neighbours does an atom in this structure have?

[4]

(e) The density of gold in the face centred cubic (fcc) structure is $1.93 \times 10^4\ kg\ m^{-3}$ and its atomic weight is $197 gmol^{-1}$.

(i) Find the separation between the close packed planes.

[4]

(ii) Assuming that the atoms are spheres which just touch one another, estimate the atomic radius.

[4]

(f) Show that the Madelung constant for a linear ionic solid having $2N$ ions of alternate charges $\pm q$ is $2ln2$.

[5]

(g) What was the physical significance of the von Laue photograph?

[2]

SECTION B

2. Consider the normal modes of a linear chain in which the force constants between nearest-neighbour atoms are alternately K and G. For equal masses and nearest-neighbour separation $a/2$, the lattice dynamics of the chain is given by

$$\begin{bmatrix} M\omega^2(k) - K - G & K + Ge^{-ika} \\ K + Ge^{+ika} & M\omega^2(k) - K - G \end{bmatrix} \begin{bmatrix} \epsilon_1 \\ \epsilon_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

(a) Show that the dispersion relations are given by

$$M\omega_{\pm}^2(k) = \left(K + G \pm \sqrt{K^2 + G^2 + 2KGcos(ka)}\right)$$

[5]

(b) Find $\omega_{\pm}(k)$ at $k = 0$ and $k = \pi/a$ for the case $K = 10G$. Sketch the dispersion relation.

[5]

(b) Discuss the dispersion relation and the Brillouin zone the limit $G \to K$.

[5]

(c) Describe what happens when you take the limit $K \to \infty$.

[5]

3. (a) Distinguish between the linear absorption coefficient (μ) and the mass absorption coefficient (μ_m) of a material. Describe the way in which μ_m varies with the wavelength of the incident X-rays.

[6]

(b) The K-absorption edge of molybdenum (Mo) is at 0.6198 Å.

(i) Find the energy (in keV) required to remove a K electron from a Mo atom.

[2]

(ii) What is the corresponding potential difference that must be applied across an X-ray tube to excite K spectra of Mo?

[2]

(iii) If the L absorptiom edge of for Mo is at 4.912 Å estimate the wavelength of the K_α line.

[4]

(c) Find the CrK_α linear absorption coefficient for chromite ($FeCr_2O_4$) with a density of 5.05 gcm^{-3}, using the physical data provided below:

Element	Atomic weight (g)	μ_m (cm^2g^{-1})
Fe	55.85	115
Cr	52.00	89.9
O	16.00	40.1

[4]

(d) What thickness of chromite powder (packing fraction 0.54) would be required if only 1 % of an incident beam of CrK_α X-rays is to be transmitted?

[2]

4. (a) Find the atomic packing fraction for a simple cubic (sc) structure.

[4]

(b) Sketch part of a simple cubic (sc) structure and indicate on it the (110) and (111) planes.

[2]

(c) For such a lattice of spherical atoms, of radius $0.16\ nm$, find the areal density $(atoms/m^2)$ on each of the planes in part (b).

[8]

(d) A second type of spherical atom just fits into the centre of each cubic cell. Find its radius and the atomic packing fraction of the modified structure.

[6]

5. (a) Discuss models for the different types of bonds that form stable molecules.

[8]

(b) A particle of mass m moves in one-dimensional motion through a field for which the potential energy of the particle–field system is

$$U(x) = \frac{A}{x^3} - \frac{B}{x}$$

where A and B are constants.

(i) Find the equilibrium position x_0 of the particle in terms of A and B.

[4]

(ii) Determine the depth U_0 of this potential well.

[3]

(iii) In moving along the x-axis, what maximum force toward the negative x direction does the particle experience?

[5]

6. The amplitude diffraction beam associated with a reciprocal lattice vector $\vec{G} = h\bar{b}_1 + k\bar{b}_2 + l\bar{b}_3$ is given by the expression $F_G = NS_G$ where N is the number of primitive unit cells in the crystal, and the structure factor S_G is given by

$$S_G(h,k,l) = \sum_j f_j exp\left[-2\pi i\left(hx_j + ky_j + lz_j\right)\right]$$

The sum is over all basis atoms at

$$\bar{r}_j = x_j\bar{a}_1 + y_j\bar{a}_2 + z_j\bar{a}_3$$

and f_j is the atomic form factor. In this problem you may assume $f_j = f$ is a constant.

(a) Using the conventional cubic cell as the unit cell for the face centred cubic (fcc) lattice, write down the positions $\bar{r}_j = (x_j, y_j, z_j)$ of the basis atoms. Show that the amplitude of a diffraction beam will be zero unless (hkl) are either all odd or all even.

[10]

(b) What is the relative spacing of the (110) planes in the fcc lattice compared to a simple cubic (sc) lattice with the same cubic unit cell size? Use this to explain why the (110) diffraction beam of the sc lattice ($n = 1$) has an amplitude of zero for the fcc lattice.

[10]

END OF PAPER