BINDURA UNIVERSITY OF SCIENCE EDUCATION SFM213

HBScSFM

FINANCIAL TIME SERIES ANALYSIS

Time : 3 hours NOV 2024

Candidates may attempt ALL questions in Section A and at most TWO questions in Section B. Each question should start on a fresh page.

SECTION A (40 marks)

Candidates may attempt ALL questions being careful to number them A1 to A4.

A1. Two main types of price changes are used in Financial Time Series; arithmetic and geometric returns (Jorion, 1997). There seems to be some confusion about the two terms, in the literature as well as among practitioners.

(a) Distinguish the two. [8]

(b) What is the significance of each? [2]

A2. (a) A commonly used model in finance is the random walk. Define the random walk process. [4]

(b) Define the stationarity of a sequence of random variables $\{X_t\}$. [4]

(c) Comment on the stationarity of and AR(1) Model and a random walk process. [2]

A3. The fundamental idea of the GARCH(1,1)-model (Bollerslev, 1986) is to describe the evolution of the variance σ_t^2 as

$$\sigma_t^2 = a_0 + a\epsilon_{t-1}^2 + b\sigma_{t-1}^2.$$

(a) State the conditions that the parameters a and b must satisfy. [3]

(b) Under what condition is the variance process stationary? Hence give an expression in terms of a_0, a and b for the stationary variance? [2]

(c) The parameter $\eta = a + b$ is known as persistence and defines how slowly a shock in the market is forgotten. Illustrate this phenomenon considering the expected size of the variance σ_k^2, k time units ahead, given the present value σ_0^2. [5]

A4. Consider the following set of data:
$\{23,32;\quad 32,33;\quad 32,88;\quad 28,98;\quad 33,16;\quad 26,33;\quad 29,88;\quad 32,69;\quad 18,98;$
$21,23;\quad 26,66;\quad 29,89\}$

(a) Calculate the lag-one sample autocorrelation of the time series. [5]

(b) Calculate the lag-two sample autocorrelation of the time series. [5]

SECTION B (60 marks)

Candidates may attempt TWO questions being careful to number them B5 to B7.

B5. (a) Suppose that r_t follows the model $r_t = r_{t-1} + a_t - 0.9a_{t-1}$; and we have $r_{1001} = 1.2$ and $r_1000(1) = 1.0$, where $r_t(1)$ denotes the 1- step ahead prediction of r_{t+1} at the forecast origin t. Compute $r_{1001}(1)$. [5]

(b) Consider a call option contingent on Stock A, which pays no dividend. Suppose that we have $c_t = \$1.12$, $P_t = 30$, $K = 29$, $r = 3\%$ per annum, and the time to expiration is $T - t = 0.25$. Is there any arbitrage opportunity? Why? [4, 1]

(c) What are the two critical assumptions used by RiskMetrics to justify the square root of time rule? [3]

(d) Consider the time series model $(1 - 0.7B + 0.8B2)r_t = 0.3 + (1 - 0.5B)a_t$, where $a_t \sim iidN(0; 1)$. Is the model stationary? Why? [3, 2]

(e) Consider the daily log returns of Coke (KO) from January 4, 2004 to April 27, 2017 for 3351 observations.

(i) An EGARCH model in the form

$$ln(\sigma_t^2) = \omega + \alpha_1 \epsilon t - 1 + \gamma_1|\epsilon_{t-1}| - 0.798) + ln(\sigma_{t-1}^2)$$

is entertained, where $a_{t-1} = \sigma_{t-1}\epsilon_{t-1}$ and $\epsilon_t \sim iidN(0, 1)$. Write down the fitted model. [4]

(ii) Based on the fitted model, is the leverage effect significant? Why? [2]

(iii) Based on the fitted model, compute the ratio $\dfrac{\sigma_t^2(\epsilon_{t-1=-3)}}{\sigma_t^2(\epsilon_{t-1=3)}}$.

What is the implication of the ratio? [4, 2]

B6. (a) Let r_t denote the daily log return of an asset.

(i) Describe a procedure for testing the existence of serial correlations in r_t and state the reference distribution of the test statistic used? [3]

(ii) Let $\mu_t = E(r_t|F_{t-1})$, where F_{t-1} denotes the information available at time $t - 1$. Write the return as $r_t = \mu_t + a_t$. Describe the null hypothesis for testing the ARCH effect of r_t, including definition of the statistics involved in H_0. [4]

(iii) Let $a_t = \sigma_t \epsilon_t$, where $\sigma^2 = E(a_t^2|F_{t-1})$ and ϵ_t are iid random variate with mean zero and variance 1. Describe a statistic discussed in class for testing the null hypothesis that ϵ_t is normally distribution. What is the reference distribution of the test statistic? [3]

(iv) Suppose that σ^2 above satisfies the model $\sigma_t^2 = 0.01 + 0.1a_{t-1}^2 + 0.8\sigma_{t-1}^2$. Compute $E(a_t)$ and $Var(a_t)$. [3, 4]

(b) Provide two reasons that may lead to serial correlations in the observed asset returns even when the underlying true returns are serially uncor- related. [2]

(c) Provide two methods that can be used to specify the order of an au- toregressive time series. [2]

(d) Describe two statistics that can be used to measure dependence between variables. [3]

(e) Provide two volatility models that can be used to model the leverage effect of asset returns. [2]

(f) Describe a nice feature and a drawback of using GARCH models to modeling asset volatility. [2]

(g) Give two potential impacts on the linear regression analysis if the serial dependence in the residuals is overlooked. [2]

B7. Consider the daily log returns of Google stock for a period with 2705 observations. Analysis of the return via R is attached. Use the output to answer the following questions.

(a) Are there serial correlations in the daily log returns? Why? [2]

(b) Is the expected log return different from zero? Why? [2]

(c) Is the distribution of the log return skew? Perform a test to justify your answer. [3]

(d) A Gaussian GARCH(1,1) model, called m1, is fitted. Write down the fitted model. [2]

(e) A Student-t GARCH(1,1) model, called m2, is also fitted. Write down the ftted model. [2]

(f) Let v denote the degrees of freedom of a standardized Student- t distribution. Based on the fitted model m2, test $H_0 : v = 5$ versus $H_1 : v \neq 5$ and draw your conclusion. [2]

(g) A skew Student-t GARCH(1,1) model, called m3, is fitted. Based on the fit, is the distribution of the standardized residuals sym- metric? Perform a test to justify your answer. [3]

(h) A TGARCH(1,1) model with Student-t distributions is also considered, called m4. Write down the fitted model. [3]

(i) Based on the fitted TGARCH model, is the leverage effect statistically significant? Why? [3]

(j) Among the fitted models, m1, m2, m3, m4, which model is preferred? Why? [2]

(k) For numerical stability, returns are multiplied by 100, i.e. in percentages. A GARCH-M model is fitted, called m5. Is the risk premium statistically signigicant? Why? [2]

(l) Let $a_t = (r_t - mean(r_t)) \times 100$ be the residuals of the mean equation for the returns. An IGARCH(1,1) model is fitted to at, called m6. Write down the fitted model. [2]

(m) Based on the fitted IGARCH(1,1) model, compute the 1-step to 3-step ahead volatility forecasts of the log return at forecast origin $t = 2705$. [2]

END OF QUESTION PAPER