BINDURA UNIVERSITY OF SCIENCE EDUCATION

FACULTY OF SCIENCE AND ENGINEERING

DEPARTMENT OF STATISTICS AND MATHEMATICS

MTE1101: ENGINEERING MATHEMATICS I

DURATION: 3 HOURS **TOTAL MARKS: 100**

JUN 2025

INSTRUCTIONS TO CANDIDATES

Answer all questions in Section A and any Two in Section B
The number of marks is indicated in brackets at the end of each question
Each question should start on a fresh page correctly numbered

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SECTION A

A1. (a) Solve the following inequality $2|x| > 3x - 10$

(b) Show that $|a + b| \leq |a| + |b|$ for all $x, y \in R$ [4,4]

A2. (a) Define what is meant by saying that l is the limit of the sequence U_n. [2]

(b) Evaluate each of the following limits

$$(i) lim_{n\to\infty}(\sqrt{n+10} - \sqrt{10})$$

$$(ii). lim_{n\to\infty} 3\sqrt{\frac{(3-\sqrt{n}\ )(\sqrt{n}+2)}{8n-4}}$$ [3,3]

A3. (a) Estimate the value of $\tan^{-1}(1.1)$ using diffferentials [4]

(b) Prove , by mathematical induction ,that , the rule of exponents is very true for every natural number n

$$(ab)^n = a^n b^n$$ [5]

A4 (a) Define what is meant by saying that l is the limit of a function f as x tends to x_0 [2]

(b) if $y(x) = y_1(x)y_2(x) \ldots y_n(x)$, use the logarithmic differentiation or otherwise to show

That $\frac{1}{y}\frac{dy}{dx} = \frac{1}{y_1}\frac{dy_1}{dx} + \frac{1}{y_2}\frac{dy_2}{dx} + \cdots + \frac{1}{y_n}\frac{dy_n}{dx}$ [4]

A5 Find each of the following integrals.

(i) $\int x^2 e^{\sin x^3} cos x^3 \; dx$ (ii) $\int x^m \; ln x \, dx$ (iii) $\int \sin^3 x \cos^3 x \, dx$ [3,3,3]

SECTION B [60 MARKS]

Candidates may attempt TWO questions being careful to number them B5 to B8

B6 (a)i. Define, in terms of ϵ and δ , what is meant by saying that a function f is continuous at a point $x = x_0$ [2]

(ii) Let $f: R \rightarrow R$ be the function defined by

$$f(x) = \begin{cases} ax - 3 & x < 2 \\ 3 - x - 2x^2 & 2 \leq x \leq 4 \\ bx + 7 & x > 4 \end{cases}$$

Where a and b are constants. If f is continuous on R , find the values of a and b [6]

(b) A sequence $\{u_n\}$ is defined by $u_{n+1} = \frac{1}{4}(2u_n + 3), u_1 = 1$

(i) Show by induction that the sequence $\{u_n\}$ is bounded above by 2 [3]

(ii) Show by induction that $\{u_n\}$ is monotonic increasing [4]

(iii) State why the sequence converges. [1]

(iv) Find the limit of the sequence [3]

(c) Show that if $0 < a < b$, *then*

$$\frac{b-a}{\sqrt{1-a^2}} < \sin^{-1}(b) - \sin^{-1}(a) < \frac{b-a}{\sqrt{1-b^2}}$$ [6]

(d) Evaluate $\int \frac{dx}{5+3cosx}$ [5]

B7 (a) (i) Use the Mean Value Theorem to prove the following inequality

$$\frac{x-1}{x} < lnx < x - 1, for \; x > 1$$ [5]

(ii)Suppose that f is continuous on [a,b] , differentiable on (a,b) and that $f'(x) > 0$ for all $\in (a,b)$.Prove that f is strictly increasing on [a,b] [4]
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(b) (i) Find $\frac{d^2y}{dx^2}$ $if \; x = 2sint \; and \; y = cos2t$ [5]

(ii) If $y = x^{\sqrt{x}}$, $x > 0$. *Find* $\frac{dy}{dx}$ [4]

(c)Evaluate each of the following limits.

(i) $\lim_{x \to 0} x^3 \ln x$

(ii) if $\lim_{x \to 0} x^{1-\cos x}$ [3,3]

(c) Show that if a function f is differentiable at a point $x = a$, then it is a continuous at that point. [6]

B8 (a)(i) Define what is meant by $\lim_{x \to \infty} u_n = \infty$ [3]

(ii) Show using the definition that $\lim_{x \to \infty} (5n - 2) = \infty$ [4]

(b) (i) Use induction to prove that $5^{2n} - 6n + 8$ is divisible by 9 for $n = 1,2, \ldots$ [5]

(ii) Sketch the general shape of the graph of the following function showing any asymptotes and any points of intersection with the axes , $f(x) = \frac{x}{x^2-1}$ [8]

(c) Show that

$$\int_0^1 \frac{xdx}{(x+1)^2(x^2+1)} = \frac{\pi - 2}{8}$$

[10]

B9 (a) Let $l_n = \int e^x \sin^n x \, dx$

(i) Show that $n^2 + 1)l_n = n(n-1)l_{n-2} \; for \; n \geq 2$ [8]

(ii)Evaluate l_4 [6]

(b) Find the area of the region bounded by the graph of $f(x) = 2x, x = 0, x = 1$ and the x-axis by calculating the limit of the Riemann Sums [10]

(c) Show that the function $f(x) = \frac{ax+b}{cx+d}$ *is* one-to-one if $ad - bc \neq 0$.What happens when $ad - bc = 0$? [6]

END OF PAPER