Bindura University of Science Education

JAN 2025

Faculty of Science Education

Department of Science and Mathematics Education

Programmes: HBSc Ed (Mathematics)

Course: **MT303: Probability Theory and Statistic**/Probability Theory 1 AMT104/SFM113

Duration: Three hours Semester Examinations

Instructions to candidates

(i) Answer all questions in Section A and two questions from Section B.

(ii) Begin each question on a fresh page

SECTION A (40 marks)

Candidate may attempt ALL questions being careful to number them A1 to A5

A1. Define the following terms:

(a) Random experiment, [2]
(b) Sample space, [2]
(c) Event. [2]

A2. Let $X_1, X_2, \ldots$ be a random sample from the double exponential distribution given by:

$$f(x) = \frac{1}{2}e^{-|x|} \qquad -\infty < x < \infty$$

(a) Find the moment generating function of X. [4]
(b) Hence, find the mean and variance of X. [6]

A3. Show that the Binomial distribution with index n and parameter p has mean np and variance npq. [10]

A4. Let $\psi = (-\infty;\ \infty)$ be the universal set. Use De Morgan's rule to find $([0,4]\ [1,6])^c$. [4]

A5. (a) Let $X_1, X_2, X_3, \dots X_n$ be independent and identically distributed Poisson random variables each with mean μ. Show that $S = X_1 + X_2 + X_3 + \dots + X_n$ has a Poisson distribution. [5]

(b) Let $X_1, X_2, X_3, \dots X_n$ be independent and identically distributed random variables, each having parameters α and β. Find the distribution of X. [5]

SECTION B (60 Marks)

Candidates may attempt TWO questions being careful to number them B6 to B9.

B6. (a) Let X have the probability density function is given by:

$$f_X(x) = \begin{cases} 2x & 0 \le x \le b \\ 0 & otherwise \end{cases}$$

(i) Sketch the graph of $f_X(x)$. [3]

(ii) Find and sketch the cumulative frequency of X. [5]

(iii)Hence, find $P(0 < X < \frac{1}{2})$. [4]

(b) Let X be a random variable with probability mass function given by:

$$p(x) = \begin{cases} \theta(1-\theta)^{x-1} & if\ x = 1, 2, 3, \dots \\ 0 & otherwise \end{cases}$$

By differentiating with respect to θ both sides of the equation

$$\sum_{x=1}^{\infty} \theta(1-\theta)^{x-1} = 1$$

Show that the mean of the geometric distribution is given by $\frac{1}{\theta}$. [6]

(c) State and prove Bayes theorem. [12]

B7. (a) Let X and Y have the joint probability density function,

$$f_{X,Y}(x,y) = \begin{cases} kxy & x = 0, 1, 2 \quad y = 1, 2, 3 \\ 0 & otherwise \end{cases}$$

(i) Find the value of the constant k. [2]

(ii) Compute the marginal distributions of X and Y. [4]

(iii) Calculate $P(Y > X)$. [2]

(iv) Comment on the independent status of X and Y. [2]

(v) Calculate the covariance X and Y. [6]

(b) Let X be a continuous random variable with parameter λ and probability density function given by:

$$f_X(x) = \lambda e^{-\lambda x}, \qquad x > 0.\ \lambda > 0$$

(i) Show that for any positive number s and t, $P(X > s + t | X > s) = P(X > t)$. [4]

(ii) Find λ given that $EX(X - 1) = 4$. [5]

(iii) Suppose that $P(X < 1) = P(X > 1)$, find λ and evaluate Var(X). [5]

B8. (a) State and prove the Chebyshev's inequality. [12]

(b) If $X \sim B(n, p)$

(i) Find the moment generating function of X. [4]

(ii) Hence find E(X) and Var(X). [4, 4]

(c) State and prove the Law of total probability [6]

END OF THE PAPER