BINDURA UNIVERSITY OF SCIENCE EDUCATION

FACULTY OF SCIENCE EDUCATION

SCIENCE & MATHEMATICS EDUCATION DEPARTMENT

DM007: VECTORS AND MATRICES

TIME: 3 HOURS

MAR 2024

INSTRUCTIONS

Answer **TWO** questions from Section A and **TWO** questions from Section B

SECTION A: Answer any **two** questions from this section

1. (a) Find a Matrix **X** satisfying the matrix equation below:

$$3\mathbf{X} + \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}^t = \begin{bmatrix} -2 & 3 \\ -1 & -2 \end{bmatrix}$$ **[10]**

(b) Use Cofactors to determine the inverse of the matrix below:

$$\begin{bmatrix} 2 & 1 & 1 \\ 1 & -2 & -3 \\ 3 & 2 & 4 \end{bmatrix}$$ **[15]**

2. Use the row transformations of the matrix to solve the system of equations below:

$2x + y + z = 8$

$x - 2y - 3z = 5$ **[25]**

$3x + 2y + 4z = 10$

3. Use Cramer's rule to solve the system:

$2x - y = 0$

$3x + z = 7$

$2x + 3z = 1$ [25]

SECTION B:

Answer any **two** questions from this section. Each question carries 25 Marks

4.

Relative to a fixed origin O, the point A has position vector $\begin{pmatrix} -2 \\ 4 \\ 7 \end{pmatrix}$

and the point B has position vector $\begin{pmatrix} -1 \\ 3 \\ 8 \end{pmatrix}$

The line l_1 passes through the points A and B.

(a) Find the vector $\overrightarrow{AB}$.

(b) Hence find a vector equation for the line l_1

The point P has position vector $\begin{pmatrix} 0 \\ 2 \\ 3 \end{pmatrix}$

Given that angle PBA is θ,

(c) show that $\cos\theta = \frac{1}{3}$

The line l_2 passes through the point P and is parallel to the line l_1

(d) Find a vector equation for the line l_2

5.

Points A (2, 2, 5), B (1, −1, −4), C (3, 3, 10) and D (8, 6, 3) are the vertices of a pyramid with a triangular base.

(i) Calculate the lengths AB and AC, and the angle BAC.

(ii) Show that $\overrightarrow{AD}$ is perpendicular to both $\overrightarrow{AB}$ and $\overrightarrow{AC}$.

(iii) Calculate the volume of the pyramid $ABCD$.

[The volume of the pyramid is $V = \frac{1}{3} \times$ base area $\times$ perpendicular height.]

6.

Relative to a fixed origin O, the point A has position vector $(10\mathbf{i} + 2\mathbf{j} + 3\mathbf{k})$, and the point B has position vector $(8\mathbf{i} + 3\mathbf{j} + 4\mathbf{k})$.

The line l passes through the points A and B.

(a) Find the vector $\overrightarrow{AB}$.

(b) Find a vector equation for the line l.

The point C has position vector $(3\mathbf{i} + 12\mathbf{j} + 3\mathbf{k})$.

The point P lies on l. Given that the vector $\overrightarrow{CP}$ is perpendicular to l,

(c) find the position vector of the point P.

END OF PAPER