BINDURA UNIVERSITY OF SCIENCE EDUCATION

FACULTY OF SCIENCE EDUCATION

Diploma in Science Education Part 1.1

DM001: Algebra Duration 3 hours

JUN 2023

Semester Examinations

INSTRUCTIONS

Answer all questions in Section A and any **two** questions from Section B

Section A: (40 marks)

A1. (a). Find k for which the equation $3x^2 + kx + 12 = 0$ has real distinct roots. **[5]**

(b). Solve the equation $2(2^x)^2 - 5(2^x) + 2 = 0$. **[6]**

A2. (a) Express $f(x) = \frac{5}{(x+1)(x^2+4)}$ in partial fractions. **[7]**

(b).) Solve the inequality; $5x^2 > 3x + 2$. **[4]**

A3. Determine whether the function $f(x) = \frac{x+2}{3x+1}$ is onto. **[5]**

A4. Let $g(x) = \frac{x-1}{2x+3}$, find g^{-1}. **[5]**

A5. (a) Show that $a^3 - b^3 \equiv (a-b)(a^2 + ab + b^2)$. **[4]**

(b). Hence, factorize completely the expression $8\,x^3 - 27y^3$. **[4]**

Section B [60 marks]

Answer **two** questions from this section being careful to number them **B6** to **B8.**

B6. (a) (i) Find D_f and R_f if f is given by $f(x) = \frac{1}{\sqrt{x-3}}$. **[5]**

(ii). When $x^3 + ax^2 - 3x + 15$ is divided by $x - 2$, the remainder is 1, find a **[6]**

(b). The roots of the equation $2x^2 - 4x + 5 = 0$ are α and β. Find the value of :

(i) $\frac{1}{\alpha} + \frac{1}{\beta}$ (ii) $\alpha^2 + \beta^2$ **[8]**

(c). Determine whether the function $f(x) = x - \frac{1}{x}$ is odd or even. [3]

(d).(i) Express $f(x) \equiv \frac{x^2}{x^2-1}$ in partial fractions. [4]

(ii) Determine the greatest or least value of the function $g(x) = 3 - 2x - x^2$. [4]

B7. (a) Solve the inequality $|3x + 1| > x$ [6]

(b) Solve the equation $\sqrt{7 - 3x} = x + 11$ [8]

(c) Find the range of values of x for which $(x - 4) < x(x - 4) \leq 5$. [10]

(d) Show that $(x^2 + 1)^{\frac{1}{2}} - x^2(x^2 + 1)^{-\frac{1}{2}} = \frac{1}{\sqrt{x^2+1}}$ [6]

B8. (a) (i). Prove the identity $\log_a b \equiv \frac{1}{\log_b a}$. [5]

(ii) Hence, solve the equation $\log_3(x + 1) - \log_9(x - 5) = 1\frac{1}{2}$. [6]

(b) Given that $2^x - 2^{-x} = 4$.

(i) Solve the equation for x, [7]

(ii). Hence, show that $|2^x + 2^{-x}| = 2\sqrt{5}$. [5]

(c) Show that if one of the roots of $x^2 - 2px + q = 0$ is twice the other then $8p^2 = 9q$. [7]

END OF PAPER