BINDURA UNIVERSITY OF SCIENCE EDUCATION

BSc Statistics and Financial Mathematics

SFM 421: General Linear Models

OCT 202

Time: 3 hours

SECTION A (40 Marks)

Candidates may attempt ALL questions being careful to number them Q1 to Q5

1. Define the following expressions:
   (a) Idempotency [3]
   (b) Best Unbiased linear estimator. [3]
   (c) Model Adequacy. [3]

2. Let $Y = X\beta + e$ where X is an $n \times p$ matrix of full rank, β is a $p \times 1$ vector of unknown parameters and e is an $n \times 1$ normally distributed random vector with mean 0 and variance $\sigma^2 I$.
   (a) Prove that $\hat{\beta}$ and $\frac{SS_{Res}}{\sigma^2}$ are independent. [5]
   (b) Show that $\frac{(n-p)S^2}{\sigma^2} = \frac{SS_{Res}}{\sigma^2}$ follows a Chi square distribution with n-p degrees of freedom. [5]
   (c) Prove tht $\frac{SS_{Res}}{n-p}$ is an unbiased estimator of σ^2. [10]

3. Explain the meaning of 70 : 30 training testing ratio. [2]

4. Explain the meaning of sufficient estimator in statistics. [2]

5. Let $X_1, X_2, ...X_n$ denote a random sample from normal distribution with mean μ and variance 1. Find the sufficient estimator for μ. [7]

SECTION B (60 Marks)

Candidates may attempt **TWO** questions being careful to number them Q6 to Q8

6. An experiment was conducted to estimate the demand for cars (Y) based on cost (X_1) and employment rate (X_2). The data obtained is presented below.

Y	X_1	X_2
1	9	17
2	10	18
3	11	19
4	12	20
5	13	21
6	14	22
7	15	23
8	16	24

(a) Find $\hat{\beta}$ [10]

(b) Construct the ANOVA table using the uncorrected sum total. [10]

(c) Test if the model is adequate. [5]

(d) Calculate $R(\beta_1/\beta_0)$. [5]

7. (a) It is known that advertising (x) influences sales (Y). Data on X and Y is presented in the table below.

X	10	9	11	12.1	14	13	7	15	16	14.2
Y	15.1	11	12	13	16	12.5	9	16.3	17	15

i. Find the 90% joint confidence interval for β assuming $\beta_0$and β_1 are significant. [10]

ii. Find the 90% confidence interval for the mean sales with advertising expenditure of 60. [5]

(b) Let $Y = X\beta + e$ where X is an $n \times p$ matrix of full rank, β is a $p \times 1$ vector of unknown parameters and e is an $n \times 1$ normally distributed random vector with mean 0 and variance $\sigma^2 I$. Show that $t'\hat{\beta}$ is BLUE for $t'\beta$ [15]

8. Let $Y = XB + e$ where X is an $n \times p$ matrix of full rank, β is a $p \times 1$ vector of unknown parameters and e is an $n \times 1$ normally distributed random vector with mean 0 and variance $\sigma^2 I$. Assume that $t'\beta$ is estimable where t is a $1 \times p$ non zero vector of real numbers and that $\hat{\beta}$ denotes any solutions to the normal equations.

(a) Show that $E(t'\hat{\beta}) = t'\beta$. [6]

(b) Prove that $Var(t'\hat{\beta}) = t'(X'X)^{-1}t\sigma^2$ [6]

(c) Deduce that the random variable $\frac{t'\hat{\beta}-t'\beta}{s\sqrt{t'(X'X)^{-1}t}}$ follows student t distribution with $n-p$ degrees of freedom. [10]

(d) Deduce that the random variable $\frac{t'\hat{\beta}-t'\beta}{\sigma\sqrt{t'(X'X)^{-1}t}}$ follows standard normal distribution. [8]