NOV 2024

# BINDURA UNIVERSITY OF SCIENCE EDUCATION

## STATISTICS AND MATHEMATICS

## MTE1101: ENGINEERING MATHEMATICS 1

Time: 3 hours

Candidates may attempt any **THREE** questions. Each question should start on a **fresh** page.

1. (a) Let $A$ and $B$ be events and let $A\Delta B = (A \cap B^c) \cup (B \cap A^c)$. Prove that:

   i. $P(A\Delta B) = P(A) + P(B) - 2P(A \cap B)$. [3]

   ii. $P(A \cup B) = P(A) + P(B) - P(A \cap B)$. [3]

   (b) Use Gaussian elimination to find the rank of the matrix $\mathbf{A}$ and a solution to the following system of inhomogeneous simultaneous linear equations, $\mathbf{Ax} = \mathbf{b}$.

$$\begin{aligned} 2x_1 + x_2 + 2x_3 + x_4 &= 5 \\ 4x_1 + 3x_2 + 7x_3 + 3x_4 &= 8 \\ -8x_1 - x_2 - x_3 + 3x_4 &= 5 \\ 6x_1 + x_2 + 2x_3 + x_4 &= 1 \end{aligned}$$

[8]

   (c) Evaluate the following limits:

   i.
$$\lim_{x \to 0} \frac{\sqrt{x+1} - 1}{x}$$ [3]

   ii.
$$\lim_{x \to +\infty} \frac{4x^3 + 1}{3x^4 + x^3 + 1}$$ [3]

2. (a) Solve the following inequalities:

   i.
$$\frac{2x - 5}{x - 2} < 1$$ [3]

   ii.
$$\frac{1}{|2x - 3|} > 5$$ [3]

   iii.
$$\frac{2}{|4x - 7|} < 2$$ [3]

(b) The functions $f$, $g$, and $h$ are defined by:

$$f(x) = 2 - x$$

$$g(x) = \frac{3}{x+1}$$

($x \neq 0$) and,

$$h(x) = 2x - 1$$

i. Show that $f^2(x) = x$. [1]

ii. Find an expression for $g^2(x)$ and state the values of $x$ for which it is defined. [1]

iii. Solve $h^3(x) = x$. [2]

(c) i. Sketch on the same diagram the graphs

$$2y = x + 1$$

and

$$2y = |x - 4|$$

[3]

ii. Solve the equations

$$2y = x + 1$$
$$2y = |x - 4|$$

[2]

(d) Given

$$A = \begin{bmatrix} 1 & 2 & 0 \\ 3 & -1 & 4 \end{bmatrix}$$

, evaluate $A^T A$ [2]

3. (a) Prove the identities:

i. $\sin^3\theta = \frac{3}{4}\sin\theta - \frac{1}{4}\sin 3\theta$ [3]

ii. $\cos 5\theta = 16\cos^5\theta - 20\cos^3\theta$ [3]

(b) Solve the equations:

i. $4e^{1+3x} - 9e^{5-2x} = 0$ [3]

ii. $3 + 2\ln(3 + \frac{x}{7}) = -4$ [3]

(c) Given $x\tan y - x^3 + \sinh(xy)$, evaluate:

i. $\frac{dy}{dx}$ [3]

ii. $\frac{d^2y}{dx^2}$ [2]

(d) Show that $\sinh^{-1} x = \ln(x + \sqrt{x^2+1})$ [3]

4. (a) Solve the differential equations:

   i.
$$\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = 0$$ [3]

   ii.
$$\frac{d^2y}{dx^2} + 4y = \cos 3x$$ [4]

(b) State the mean value theorem for integrals. [2]

(c) State and prove the second fundamental theorem of calculus. [4]

(d) Given that

$$\mathbf{u} = 3i + 2j + 5k$$

and

$$\mathbf{v} = 6i + 7j + 9k$$

Evaluate:

   i. $\mathbf{u} \cdot \mathbf{v}$ [2]

   ii. $\mathbf{u} \times \mathbf{v}$ [3]

   iii. Find a unit vector that is normal to **u** and **v** [2]

5. (a) Find the equation of the tangent to the circle $r = 6\sin\theta$ when $\theta = \frac{\pi}{3}$ [6]

(b) Referring to (a) above, at which points do we have:

   i. A horizontal tangent? [2]

   ii. A vertical tangent? [2]

(c) Evaluate the following limits:

   i. $\lim_{x\to 0} x \ln x$ [3]

   ii. $\lim_{x\to\infty} x^{\frac{1}{x}}$ [3]

(d) Solve the inequality $\dfrac{1}{x^2+2x-8} > 0$ [4]

**END OF EXAM**