BINDURA UNIVERSITY OF SCIENCE EDUCATION

HONORS DEGREE IN SCIENCE EDUCATION (HBScED)

MT304: Vector Calculus

Time: 2 hours

Candidates may attempt ALL questions in Section A and at most TWO questions in Section B. Each question should start on a fresh page.

SECTION A (40 marks)

Candidates may attempt ALL questions being careful to number them A1 to A5.

A1. (a) Distinguish between vector and scalar giving an example for each one. [4]

(b) Suppose $\overline{V} = (a, b, c)$. Find the unit vector of $\overline{V}$. [3]

(c) Find b so that $2i - 3j + 5k$ and $3i + bj - 2k$ are orthogonal vectors. [3]

A2. Define the following terms:

(a) permutation symbols, [3]

(b) linearly independent vectors. [1]

A3. Prove that $(\overline{U} \times \overline{V})_i = \epsilon_{ijk}\overline{U}_j\overline{V}_k$ using tensor analysis. [7]

A4. Use the definition of derivative of a vector function to find $\vec{G}'(t)$ where

$\vec{G}(t) = ti - t^2 j + 4tk.$ [6]

A5. (a)Find the unit vector parallel to the line $2x - 1 = 3y = 2z + 4$. [3]

(b) Show that $\frac{d\hat{N}}{dS} = \tau\hat{B} - \varkappa\hat{T}$. [10]

SECTION B (60 marks)

Candidates may attempt TWO questions being careful to number them B6 to B8

B6. (a) Given that $\vec{E}(x, y) = x^2 i + y^2 j$, show that $\overrightarrow{E_x} \cdot \overrightarrow{E_y} = 0$. [5]

(b) Find the unit normal vector for a surface S represented by $x = x, y = y, z = (x, y)$; where x and y are parameters. [7]

(c) Suppose $\lim_{t \to a} \overline{F}(t) = \vec{H}(t)$ and $\lim_{t \to a} \overline{G}(t) = \vec{K}(t)$. Prove that

$\lim_{t \to a} \overline{\{F(t) + G(t)\}} = \vec{H}(t) + \vec{K}(t)$. [6]

(d) Sketch the vector field $\overline{F} = -yi + xj$. [7]

(e) Evaluate the line integral of $\overline{F}(x, y, z)$along the path C such that

(f) $\overline{F}(t) = (\cos t,\ \sin t, 0), 0 \leq t \leq 1$. [5]

B7. (a) Find the directional derivative of $\varphi = 4e^{2x-y+z}$ at the point (1, 1, 1) in the direction towards the point (-3, 5, 6). [7]

(b) Verify Stoke's Theorem for the vector field $\vec{F} = (2x - y)i - yz^2 j - y^2 zk$ and S is the surface$x^2 + y^2 + z^2 = 1$ above the xy-plane and C its boundary. [12]

(c) Let $F(x, y, z) = F_1(x, y, z)i + F_2(x, y, z)j + F_3(x, y, z)k$. Show that

$$F_y(x, y, z) = \frac{\partial F_1}{\partial y} i + \frac{\partial F_2}{\partial y} j + \frac{\partial F_3}{\partial y} k. \quad [6]$$

(d) Show that $\nabla(F + G) = \nabla F + \nabla G$. [5]

B8. (a) If $\overline{M}(x, y) = e^{xy^2} i + (x^2 + y)j + x\cos yk$. Calculate:

(i) $\overline{M_x}$, [2]

(ii) $\overline{M_y}$, [2]

(iii) $\overline{M_{xy}}$

(iv) $\overline{M_x} \times \overline{M_y}$, [3]

(v) $\overline{M_x} \cdot \overline{M_y}$ [3]

(b) (i) Define a curl. [1]

(ii) Show that the vector field $\vec{V}(t) = 2xyi + (x^2 + z)j + yk$ is conservative. Hence find its scalar potential. [4, 3]

(c) (i) Define the term curvature. [1]

(ii) Let C be the curve represented by $\vec{r}(t) = a\cos t\, i + a\sin t\, j + btk$. Find the curvature of C. [6]

(d) For vector field $\vec{F}(t) = yi + xzj + (x + y)k$, write its cylindrical coordinate system. [5]

END OF PAPER