BINDURA UNIVERSITY OF SCIENCE EDUCATION

HBsc Statistics and Financial Mathematics

SFM 413 Financial Derivatives

Time: 3 hours

Candidates may attempt **ALL** questions in Section A and at most **TWO** questions in Section B. Each question should start on a fresh page.

SECTION A (40 marks)

Candidates may attempt ALL questions being careful to number them 1 to 3.

1. (a) Define the following terms.
   i. Option in the money. [3]
   ii. Short position. [3]
   iii. Divisibility. [3]
   iv. Arbitrage. [3]

   (b) Denote the European call option price in the Black- Scholes model by.

$$C^E = SN(d_1) - Xe^{-rT}N(d_2)$$

   where

$$d_1 = \frac{ln\frac{S}{X}+(r+\frac{1}{2}\sigma^2)T}{\sigma\sqrt{T}}$$

   Prove that the theta of an option is given by

$$\theta = \frac{-s\sigma}{2\sqrt{2\pi T}}e^{-d_1^2} - rXe^{-rT}N(d_2)$$

   [11]

2. Let $S_0 = \$50$, $r = 5\%$, $u = 0.1$ and $d = -0.1$. Find the price of a European call and put option with strike price $X = \$60$ to be exercised after $N = 3$ steps. . [9]

3. Distinguish between

(a) Sub- martingale and super martingale. [4]

(b) Hedging and Speculation. [4]

SECTION B (60 marks)

Candidates may attempt **TWO** questions being careful to number 4 to 6

4. (a) State and Prove the Coxx- Ross Rubinstein formula. [15]

   (b) A stock is currently \$50. It is expected to go up by $u = 0.1$ and down by $d = -0.1$. The risk free interest rate is 5% per annum. What is the price of the European call and put option with a strike price of \$60, to be exercised after 5 steps? Apply the CRR formula assuming periodic compounding. [15]

5. (a) Prove that for a stock paying no dividends, the forward price is $F(0;T) = S0e^{rT}$. [15]

   (b) Compute the value of an American put and call options expiring at time 3 with strike price $K = \$70$ on a stock with initial price $S_0 = \$65$ in a binomial tree model with $u = 0.1$, $d = -0.05$ and $r = 0.03$. [10]

   (c) Prove that the future value V_t increases if any one of the parameters m, t, r or P increases while others remain constant. [5]

6. (a) The Black Scholes formula has been used to price European and American options among others.

      i. What factors affect the Black Scholes formula?. [4]

      ii. State the Black Scholes formula for European put options, clearly giving meaning of each parameter. [5]

      iii. State the assumptions made in order to apply the Black Scholes formula. [4]

   (b) The stock price six months from expiration of an option is \$42. The exercise price of the option is \$40 and the risk free interest rate is 10% per annum. The stock has a volatility of 20% per annum. Calculate the price of a European call option. [7]

   (c) State and prove the Put- Call parity theorem. [10]