BINDURA UNIVERSITY OF SCIENCE EDUCATION SFM223

BACHELOR OF SCIENCE HONOURS IN STATISTICS & FINANCIAL MATHEMATICS

NUMERICAL METHODS IN FINANCE

Time : 3 hours APR 2025

Candidates may attempt ALL questions in Section A and at most TWO questions in Section B. Each question should start on a fresh page.

SECTION A (40 marks)

Candidates may attempt ALL questions being careful to number them A1 to A5.

A1. (a) Distinguish the following concepts

(i) Rounding off versus chopping off, [2]

(ii) interpolation versus extrapolation, [2]

(iii) local versus global error. [2]

(b) Define the following concepts

(i) Bit, [2]

(ii) presicion, [2]

(iii) error bound. [2]

A2. Suppose that the bisection method is started with the interval [50,63]. How many steps should be taken to compute a root with relative accuracy of one part in 10^{12} [5]

A3. Use the Newton Method to find a non-zero solution of $x = 2sinx$. [6]

A4. Find the appropriate Lagrange interpolating polynomial using the table:

x	0	0.5	1	1.5
f(x)	1	2	3	4

[7]

A5. (a) State one disadvantage of the LU Decomposition methods of solving linear equations and suggest how it can be mitigated. [2]

(b) Solve using LU Decomposition

$$x_1 + 2x_2 + 4x_3 = 3$$

$$3x_1 + 8x_2 + 14x_3 = 13$$

$$2x_1 + 6x_2 + 13x_3 = 4.$$

[8]

SECTION B (60 marks)

Candidates may attempt TWO questions being careful to number them B6 to B8.

B6. (a) Suppose that the function $f(x)$ is known at the $n+1$ points labelled as x_0, x_1, . . . , x_n, with the endpoints given by $x_0 = a$ and $x_n = b$. Derive the Composite Trapezoidal rule for the numerical quadrature. [10]

(b) Apply the Gauss-Seidel method to solve

$$\begin{aligned} 5x_1 - 2x_2 + 3x_3 &= -1, \\ -3x_1 + 9x_2 + x_3 &= 2, \\ 2x_1 - x_2 - 7x_3 &= 3. \end{aligned}$$

Continue iterations until two successive approximations are identical when rounded to three significant digits. [8]

(c) Construct the polynomial interpolating the data using Lagrange polynomials. **Table 1**

x	-1	0	1
y	1	2	-1

[10]

B7. (a) Give three reasons and explain why polynomials are a common choice for interpolating functions? [5]

(b) The upward velocity of a rocket is given as a function of time in Table 1.

Table 2: Velocity as a function of time.

t(s)	0	10	15	20
v(t)(m/s)	0	227.04	362.78	517.35

(i) Construct the Newton's divided difference table for the data. [15]

(ii) Using the polynomial interpolant for velocity, find the distance covered by the rocket from $t = 11s$ to $t = 16s$. [5]

(iii) Using the polynomial interpolant for velocity, find the acceleration of the rocket at $t = 16s$. [5]

B8. (a) Determine the parameters a, b, c, d, e, f, g and h so that $S(x)$ is a natural cubic spline, where

$$S(x) = \begin{cases} ax^3 + bx^2 + cx + d, & x \in [-1, 0], \\ ex^3 + fx^2 + gx + h, & x \in [0, 1], \end{cases}$$

with interpolation conditions $S(-1) = 1$, $S(0) = 2$ and $S(1) = -1$. Sketch the graph. [20]

(b) Use 4^{th} order Runge Kutta methods to solve $\frac{dy}{dx} = -2y + x + 4$, $y(0) = 1$ and $h = 0.2$. [10]

END OF QUESTION PAPER