BINDURA UNIVERSITY OF SCIENCE EDUCATION

FACULTY OF SCIENCE AND ENGINEERING

DEPARTMENT OF STATISTICS AND MATHEMATICS

SFM221: **OPERATIONS RESEARCH IN FINANCE** NOV 2024

DURATION: **3 HOURS**

EXAMINATION

INSTRUCTIONS TO CANDIDATES

1. Answer any **two** questions from Section A and any other **two** questions from Section B to give a total of **four** questions.
2. You may make use of a non-programmable scientific calculator.

SECTION A

QUESTION 1

M&D Chemicals produces two products that are sold as raw materials to companies manufacturing soaps and laundry detergents. Based on an analysis of current inventory levels and potential demand for the coming month, M&D's management specified that the combined production for products A and B must total at least 350 gallons. Separately, a major customer's order for 125 gallons of product A must be satisfied. Product A requires 2 hours of processing time per gallon and product B requires 1 hour of processing time per gallon. For the coming month, 600 hours of processing time are available. M&D's objective is to satisfy these requirements at a minimum total production cost. Production costs are $2 per gallon for product A and $3 per gallon for product B.

(a) Formulate M&D Chemicals problem as a linear program that can be adopted in order to minimise the cost of the production schedule. [7]

(b) Determine the minimum-cost production schedule for M&D Chemicals using the graphical method. [12]

(c) Determine the dual prices for the demand of product A, total production and processing time resource constraints. [6]

[25 Marks]

QUESTION 2

TOYCO assembles three types of toys – trains, trucks and cars- using three operations. The daily limits on the available times for the three operations are 430, 460, and 420 minutes, respectively, and the revenues per unit of toy train, truck, and car are \$3, \$2, and \$5, respectively. The assembly times per train at the three operations are 1, 3, and 1 minutes, respectively. The corresponding times per trucks and per car are (2, 0, 4) and (1, 2, 0) minutes (a zero time indicates that the operation is not used).

Letting $x_1, x_2, and\ x_3$ represent the daily number of units assembled of trains, trucks, and cars, respectively, the associated LP model is given as:

$$Maximise\ z = 3x_1 + 2x_2 + 5x_3$$

Subject to

$$x_1 + 2x_2 + x_3 \leq 430 \quad (Operation\ 1)$$

$$3x_1 + 2x_3 \leq 460 \quad (Operation\ 2)$$

$$x_1 + 4x_2 \leq 420 \quad (Operation\ 3)$$

$$x_1, x_2, x_3 \geq 0$$

Using $x_4, x_5, and\ x_6$ as the slack variables for the constraints of operations 1, 2 and 3, respectively, the optimum tableau is given as;

Table 1: Optimal Table for TOYCO Company

Basic	z	x1	x2	x3	x4	x5	x6	Solution
Z	1	4	0	0	1	2	0	1350
x2	0	- 1/4	1	0	1/2	- ¼	0	100
x3	0	3/2	0	1	0	½	0	230
x6	0	2	0	0	-2	1	1	20

(a) Determine the status of each resource. [3]

(b) Determine the dual prices for operations 1, 2 and 3 respectively. [3]

(c) Determine the feasibility ranges for the dual prices obtained in (b). [9]

(d) If the availabilities of operations 1, 2 and 3 are changed to 440, 490, and 400 minutes, respectively, determine the change in the optimal revenue using the optimal dual prices. [5]

(e) Determine the new optimal solution if the objective function of TOYCO is changed from $z = 3x_1 + 2x_2 + 5x_3$ to $z = 2x_1 + x_2 + 6x_3$. [5]

[25 Marks]

QUESTION 3

A transportation problem involves the following costs, supply, and demand as in Table 2.

Table 2: Supply, Demand and Transportation Costs per Item

	To (cost $)				
From	1	2	3	4	Supply
1	500	750	300	450	12
2	650	800	400	600	17
3	400	700	500	550	11
Demand	10	10	10	10	

(a) Formulate a balanced transportation problem that can be used to minimise the transport costs. [6]

(b) Use the following methods to determine the basic feasible solution.

i. Northwest Corner Method [3]

ii. The Minimum Cost Method [4]

iii. The Vogel's Approximation Method [4]

(c) Using the basic feasible solution obtained by the Northwest corner method, determine the optimal solution using the transportation simplex method. [8]

[25 Marks]

SECTION B

QUESTION 4

JoShop needs to assign 4 jobs to 4 workers. The cost of performing a job is a function of the skills of the workers. Table 3 summarises the costs of the assignments. Worker 1 cannot do job 3 and worker 3 cannot do job 4.

Table 3: Costs Associated By Each Worker to Perform Each Job

Worker	Cost ($)			
	Job 1	Job 2	Job 3	Job 4
1	50	50	*	20
2	70	40	20	30
3	90	30	50	*
4	70	20	60	70

Note: * indicates that the worker cannot do that particular job.

(a) Develop a linear programming model to minimise the time it takes to perform the four jobs. [10]

(b) Determine the assignment of workers to jobs that minimizes the total cost required to perform the four jobs using the Hungarian Method. [12]

(c) What is the associated optimal cost to perform the four jobs? [3]

[25 Marks]

QUESTION 5

(a) Green Bar Soap is produced on a production line that has an annual capacity of 60,000 cases. The annual demand is estimated at 26,000 cases, with the demand rate essentially constant throughout the year. The cleaning, preparation, and setup of the production line cost approximately $135. The manufacturing cost per case is $4.50, and the annual holding cost is figured at a 24% rate. Thus holding cost = IC = 0.24($4.50) = $1.08.

i. What is the recommended production lot size? [3]

ii. What is the total annual inventory cost? [3]

iii. Given that there is a five-day lead time (to schedule and set up a production run) and 250 working days/year. Calculate the reorder point. [2]

iv. Calculate the time between production runs (Cycle Time). [2]

(b) TV Sales Company has a product for which the assumptions of the inventory model with backorder are valid. Information obtained from the company is as follows:

D = 2,000 units per year, I = 20%, C = $50 per unit,

h = IC = (0.2) ($50) = $10 per unit per year and k = $25 per order.

The annual backorder cost (penalty) is estimated to be $30 per unit per year. It is assumed that a year has 250 working days and backorders are allowed.

(i) What is the optimal order quantity? [3]

(ii) Determine the maximum attainable inventory. [3]

(iii) What is the maximum shortage quantity? [3]

(iv) Calculate the cycle time. [3]

(v) If the company chooses to prohibit backorders and adopts the regular EOQ model, what will be the new economic order quantity? [3]

[25 Marks]

QUESTION 6

Extra Profile Service of Bulawayo recently purchased land and wants to determine the size of a holiday resort it should build. The sizes of holiday resorts being considered are small and large. At the same time, an uncertain economy makes it difficult to ascertain the demand for the new resort area. Extra Profile Service's management realises that a large development followed by a low demand could be very costly to the company. However, if Extra Profile Service makes a conservative small development decision and then finds a high demand, the firm's profits will be lower than they might have been. With the three levels of demand (low, medium, and high), Extra Profile Service's management has prepared the profit ($1000s) payoff matrix in Table 4.

Table 4: Extra Profile Service Payoff Table

Decision Alternatives	Demand		
	Low (**s1**)	Medium (**s2**)	High (**s3**)
Small (**d1**)	75	100	100
Large (**d2**)	25	100	250
Probability	0.65	0.15	0.20

(a) For the Maximin, Maximax and Minimax Regret criteria, determine Extra Profile Service's choice of the size of a holiday resort it should build. [9]

(b) What is the optimal decision strategy if perfect information were available? [4]

(c) What is the expected value for the decision strategy developed in part (b)? [2]

(d) Using the expected value approach, what is the recommended decision and its expected value without perfect information? [6]

(e) What is the expected value of perfect information (EVPI)? [4]

[25 Marks]

END OF EXAMINATION PAPER