BINDURA UNIVERSITY OF SCIENCE EDUCATION

DM003: Calculus

Time : 3 hours

JUN 2024

Answer ALL questions in Section A and at most TWO questions in section B.

SECTION A (40 marks)

Candidates may attempt ALL questions being careful to number them A1 to A5.

A1. (a) Find the coordinates of the stationary points of the curve $y = x^4 - 4x^3$. [4]

(b) Define a function. [3]

(c) Write down the area of a circle as a function of its radius. [2]

A2. (a) If $f(x) = x^2 + 3x$. Find:

(i) $f(1)$. [2]

(ii) $f(x + c)$. [2]

(b) Find the coordinates of the stationary points of the curve $y = x^4 - 4x^3 + 27$. [4]

A3. Determine the equation of the normal at $t = 2$ given by the parametric equations $x = \frac{3t}{1+t}$; $y = \frac{t^2}{1+t}$. [5]

A4. Determine the volume of the shape created when $y = \frac{1}{2}x$ is rotated around the x- axis from $0 \leq x \leq 4$. [7]

A5. (a) Define a Derivative. [3]

(b) Use the definition of a derivative to differentiate the function $y = 1 - x^2$. [8]

SECTION B (60 marks)

Candidates may attempt TWO questions being careful to number them B6 to B8.

B6. (a) Find the area under the curve $y = \frac{1}{x^2}$ between the lines $x = 1$ and $x = 3$. [5]

(b) Find the area of the region bounded by $y = 1 - x^2$ and the x-axis. [5]

(c) Derive the formula for the volume of a sphere of radius r . [10]

(d) State any five rules of differentiation. [5]

(e) Find the general solution of the differential equation $\frac{dy}{dx} = (1 - y)^2$, expressing y in terms of x. [5]

B7. (a) Solve the differential equation $\frac{dy}{dx}x^2 = y(y - 1)$. [5]

(b) Use Simpson's rule with six strips to estimate $\int_1^4 \sqrt{1 + x^3}dx$. [8]

(c) Find the volume of a cone swept out by the line $y = 2x$ rotated about the x-axis between $x = 0$ and $x = 5$. [4]

(d) Find the general solution of $\frac{dy}{dx} + 3x(y^2 + 4) = 0$, expressing y in terms of x. [7]

(e) Find the equation of the tangent to $f(x) = x^3 - 3x^2 + x - 1$ at the point where $x = 2$. [6]

B8. (a) Use a strip width of $\frac{\pi}{10}$ to evaluate $\int_0^{\frac{2\pi}{5}} tan(x)dx$,using the trapeium rule. [10]

(b) Find the area enclosed by the x-axis, $x = 2$ and $x = 4$ and the graph of $y = \frac{x^3}{10}$. [5]

(c) Evaluate the integral $\int_0^3 \frac{x}{1+x^2}dx$. [8]

(d) Find the area of the region bounded by $y = x^3, x = -1, x = 2$ and the x-axis. [7]

END OF QUESTION PAPER