BINDURA UNIVERSITY OF SCIENCE EDUCATION

FACULTY OF SCIENCE EDUCATION

Diploma in Science Education Part 1.1

DM008: Numerical Methods Duration 3 hours

Semester Examinations

NOV 2023

INSTRUCTIONS

Answer all questions in Section A and any **two** questions from Section B

Section A: (40 marks)

A1. Variables x and y are connected by the relation a law of the form $y = kx^n$, where k and n are constant integers.

(i) Transform the relation $y = kx^n$ to linear form. [3]

Approximate values of y corresponding to the given values of x are given in the following table.

x	1.34	3.58	7.6	12.1	14.8
y	208.0	10.9	1.14	0.283	0.155

(ii) Find, by plotting a suitable graph the values of k and n. [7]

A2. (i) Use Simpson's rule to estimate the value of $\int_0^{0.8} \sqrt{(1+x^2)}\,dx$ using ordinates at intervals of 0.2. [8]

(ii) Suggest two reasons why numerical solutions are important. [2]

A3. Let $f(x) = xe^x - x - 1$.

(a) Show that the equation $f(x) = 0$ has root between 0 and 1. [2]

(b) Taking $x_o = 1$ as an initial approximation, determine the value of the root to 3 significant figures by:

(i) fixed point iteration, [2]

(ii) Newton-Raphson method. [4]

(iii) Compare the efficiency of the two methods in b (i) and (ii). [2]

A4. (a) The chord AB of a circle divdes the circle into two portions whose areas are in the ratio 3: 1. If AB makes an angle θ with the diameter passing through A, show that θ satisfies the equation: $\sin 2\theta = \frac{\pi}{2} - 2\theta$. [5]

(b) Solve the equation $\sin 2\theta = \frac{\pi}{2} - 2\theta$ by graphical means. [5]

Section B [60 marks]

Answer **two** questions from this section being careful to number them **B5** to **B7.**

B5. (a) If x_1 is the first approximation of the root of the equation $f(x) = 0$ and x_2 is the second approximation of the root:

(i) State the underlying concept of the Newton-Raphson iterative method. [2]

(ii) Hence, show that $x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$. [4]

(iii) Apply the Newton-Raphson method to find the root of the equation $x + \frac{4}{x^2} - 1$, with $x_0 = -1$, giving your answer to 3 significant figures. [7]

(b) (i) Show that if x is a fixed point of the iteratin then, $x_{n+1} = \sqrt{3x_n + 2}$, then x satisfies the equation: $x^2 - 3x - 2 = 0$.

(iii) Perform 4 iterations of for $x_{n+1} = \sqrt{3x_n + 2}$, using $x_0 = 1$ to obtain x_4. [6]

(c). (i) Use linear interpolation to find the root of the equation $e^x = 3x + 1$ to 3 decimal places. [5]

(ii) It is known that x and y are relaed by the law $ae^y = x^2 - bx$. Explain how you would reduce the relation to the form $Y = mX + c$. [6]

B 6. (a) (i). Sketch the graph of the function $f(x) = 8 - 3x - 2x^2$ [4]

(ii) Show by differentiation that the gradient of the function . [2]

(iii) Use the iteration $x_{n+1} = \sqrt[3]{4 - 1.5x_n}$ to find the root of the equation $8 - 3x - 2x^2 = 0$ correct to 3 significant figures. [7]

(b) (i) Estimate to 4 decimal places, the definite integral $I = \int_0^1 \frac{1}{1+x^2} dx$ using Simpson's rule with 5 ordinates. [7]

(ii) Use analytic methods to find $\int_0^1 \frac{1}{1+x^2} dx$ and hence determine the relative error in using Simpson's rule. **[10]**

B7. (a) (i). If $I = \int_a^b f(x)dx$ then prove that $I = \frac{b-a}{n}[y_0 + y_n + 2(y_1 + y_2 + \cdots + y_{n-1})]$, where n denotes the number of strips. **[6]**

(ii). Estimate the value of $\int_1^e lnxdx$ using the trapezium rule with 5 ordinates to 4 decimal places. **[7]**

(iii). Hence, determine the relative error in the trapezium rule. **[3]**

(b) The sequence given by the iterative formula: $x_{n+1} = 2(1 + e^{-x_n})$ with $x_0 = 0$, converges to the root, α. Find, an estimate for α to 3 decimal places and state the equation which α is a root. **[7]**

(c) (i) Verify that the equation $x^3 - 7x + 3 = 0$ has a root between 2 and 3. **[3]**

(ii). Use the interval bisection method to determine an approximation of the root in (i) to 3 decmal places. **[5]**

END OF PAPER