BINDURA UNIVERSITY OF SCIENCE EDUCATION

FACULTY OF SCIENCE AND ENGINEERING

DEPARTMENT: ENGINEERING AND PHYSICS

PROGRAMME: BACHELOR OF SCIENCE EDUCATION HONOURS DEGREE IN PHYSICS (HBScEdPH)

COURSE CODE (s) HPH 111/PH 101 (1): MECHANICS AND OSCILLATIONS

DURATION: 3 HOURS **TOTAL MARKS: 100**

INSTRUCTIONS TO CANDIDATES

Answer **ALL** questions in Section A and any **THREE** questions from Section B. Section A carries 40 marks and Section B carries 60 marks.

Physical Constants

Gravitational Constant	G	$6.67 \times 10^{-11}\, m^3\, kg^{-1} s^{-2}$
Gravitational Acceleration	g	$9.80\, ms^{-2}$
Speed of Light	c	$3 \times 10^8\, m\, s^{-1}$
Planck's Constant	h	$6.62607015 \times 10^{-34}\, J\, s$
Boltzmann Constant	k_B	$1.380649 \times 10^{-23}\, J\, K^{-1}$

Section A

1(a) (i) Describe the difference between average speed and instantaneous speed. (2 Marks)

(ii) A car accelerates uniformly from rest to a speed of 25 m/s in 10 seconds. Calculate the acceleration and the total distance covered. (4 Marks)

(b) (i) Explain how projectile motion can be analyzed as two independent motions in horizontal and vertical directions.

(4 Marks)

(ii) A projectile is launched with an initial velocity of 20 m/s at an angle of 30° to the horizontal. Calculate the maximum height and range of the projectile.

(7 Marks)

(c) A 5 kg block is pulled across a frictionless surface with a constant force of 20 N. Calculate the acceleration of the block.

(3 Marks)

(d) Define linear momentum and state its formula.

(2 Marks)

(e) Define work and calculate the work done by a force of 50 N acting over a distance of 10 meters at an angle of 60° to the direction of motion.

(4 Marks)

(f) (i) Define simple harmonic motion and derive the equation for the period of a simple pendulum.

(6 Marks)

(ii) A mass-spring system oscillates with a frequency of 2 Hz. Calculate the spring constant if the mass is 0.5 kg.

(4 Marks)

(g) (i) Explain the difference between angular velocity and angular acceleration. Give an example of each.

(4 Marks)

(ii)A rotating disk accelerates uniformly from rest to an angular velocity of 10 rad/s in 5 seconds. Calculate the angular acceleration and the total angle rotated.

(4 Marks)

Section B

2 A ball is thrown upward from the top of a building with an initial velocity v_0 $20\,m/s$. The building is 40 m high and the ball just misses the edge of the building roof on its way down; see Figure. 2.1 and take $g = \mathbf{10}\,\boldsymbol{m/s^2}$. Neglecting air resistance, find:

(a) the time t_1 for the ball to reach its highest point,

(2 Marks)

(b) how high will it rise,

(3 Marks)

(c) how long will it take to return to its starting point

(6 Marks)

(d) the velocity v_2 of the ball at this instant, and

(3 Marks)

(e) the velocity v_3 and the total time of flight t_3 just before the ball hits the ground

(6 Marks)

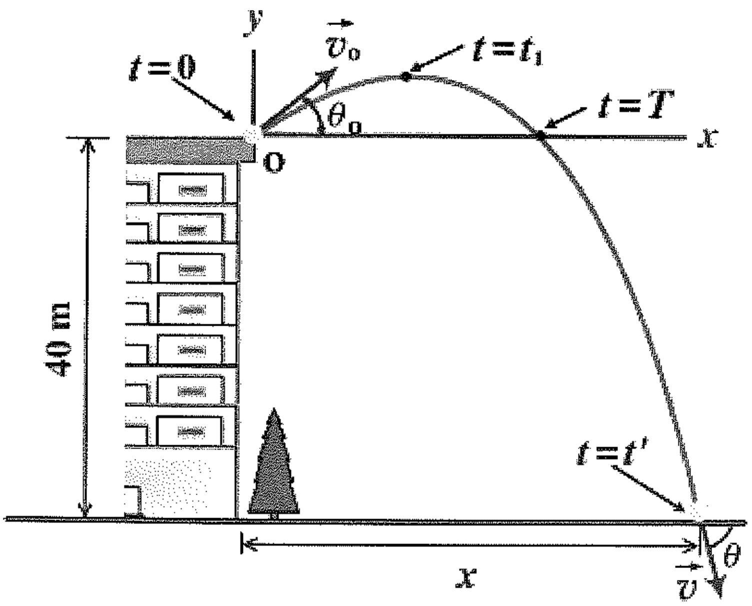


Figure 2.1

3. In a cathode ray tube of a TV set, an electron with initial velocity

$v_0 = 2 \times 10^4\ m/s$ enters a region 2 cm long (see Figure. 3.1) where it is electrically accelerated in a straight line. The electron emerges from this region with a velocity $v = 3 \times 10^5\ m/s$.

(a) What was its acceleration, assuming it was constant?

(8 Marks)

(b) How long will the electron be in this region?

(12 Marks)

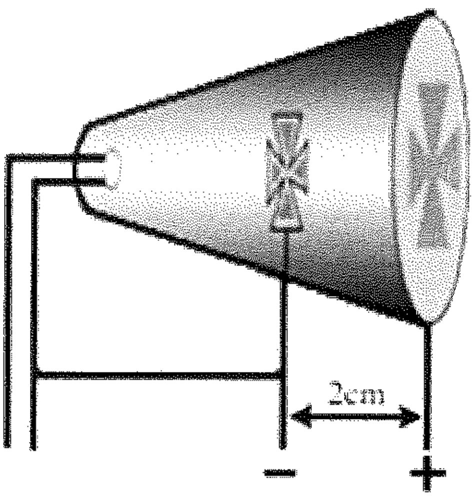


Figure 3.1

4. Figure 4.1 shows the following three vectors:

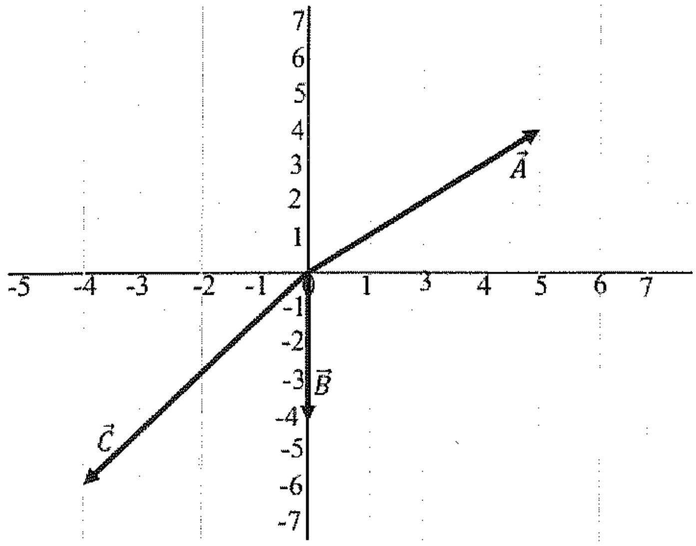


Figure 4.1

Find

(a) the vector sum $\vec{R}$ of these three vectors?

(10 Marks)

(b) the magnitude of $\vec{R}$?

(5 Marks)

(c) the angle measured from the $+x$ direction, (θ) ?

(5 Marks)

5 (a) A block of mass $m = 21\ kg$ hangs from three cords as shown in part (a) of Figure. 5.1. Taking $sin\ \theta = 4/5$, $cos\ \theta = 3/5$, $sin\emptyset = 5/13$, and $cos\ \emptyset = 12/13$, find the tensions in the three cords.

(15 Marks)

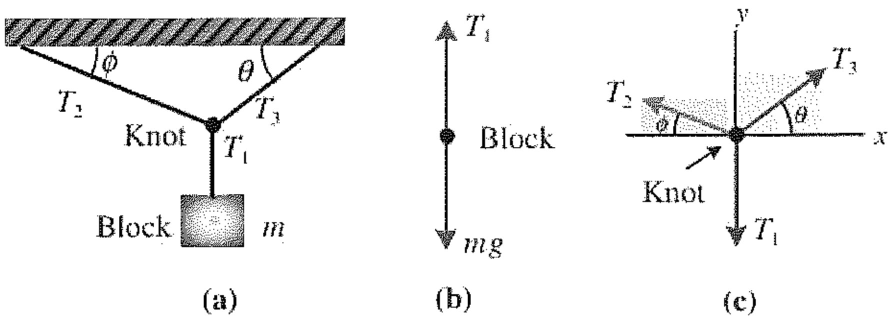


Figure 5.1

(b) Discuss frictional force.
(5 Marks)

6 A particle oscillates with a simple harmonic motion along the x axis. Its displacement from the origin varies with time according to the equation:

$$x = (2\,m)\,cos(0.5\pi\,t + \pi/3)$$

where t is in seconds and the argument of the cosine is in radians

(a) Find the amplitude, frequency, and period of the motion.
(4 Marks)

(b) Find the velocity and acceleration of the particle at any time
(6 Marks)

(c) Find both the maximum speed and acceleration of the particle.
(4 Marks)

(d) Find the displacement of the particle between $t = 0$ and $t = 2s$.
(6 Marks)

END OF PAPER

Mechanics and Oscillations Formula Sheet

1 MECHANICS

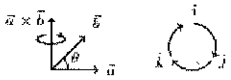
1.1: Vectors

Notation: $\vec{a} = a_x\,\hat{\imath} + a_y\,\hat{\jmath} + a_z\,\hat{k}$

Magnitude: $a = |\vec{a}| = \sqrt{a_x^2 + a_y^2 + a_z^2}$

Dot product: $\vec{a}\cdot\vec{b} = a_x b_x + a_y b_y + a_z b_z = ab\cos\theta$

Cross product:



$$\vec{a}\times\vec{b} = (a_y b_z - a_z b_y)\hat{\imath} + (a_z b_x - a_x b_z)\hat{\jmath} + (a_x b_y - a_y b_x)\hat{k}$$

$$|\vec{a}\times\vec{b}| = ab\sin\theta$$

1.2: Kinematics

Average and Instantaneous Vel. and Accel.:

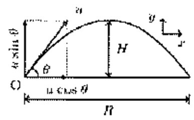
$$\vec{v}_{\rm av} = \Delta\vec{r}/\Delta t, \qquad \vec{v}_{\rm inst} = d\vec{r}/dt$$
$$\vec{a}_{\rm av} = \Delta\vec{v}/\Delta t \qquad \vec{a}_{\rm inst} = d\vec{v}/dt$$

Motion in a straight line with constant a:

$$v = u + at, \quad s = ut + \tfrac{1}{2}at^2, \quad v^2 - u^2 = 2as$$

Relative Velocity: $\vec{v}_{A/B} = \vec{v}_A - \vec{v}_B$

Projectile Motion:



$$x = ut\cos\theta, \quad y = ut\sin\theta - \tfrac{1}{2}gt^2$$
$$y = x\tan\theta - \frac{g}{2u^2\cos^2\theta}x^2$$
$$T = \frac{2u\sin\theta}{g}, \quad R = \frac{u^2\sin 2\theta}{g}, \quad H = \frac{u^2\sin^2\theta}{2g}$$

1.3: Newton's Laws and Friction

Linear momentum: $\vec{p} = m\vec{v}$

Newton's first law: inertial frame.

Newton's second law: $\vec{F} = \frac{d\vec{p}}{dt}$, $\vec{F} = m\vec{a}$

Newton's third law: $\vec{F}_{\rm AB} = -\vec{F}_{\rm BA}$

Frictional force: $f_{\rm static,\,max} = \mu_s N$, $f_{\rm kinetic} = \mu_k N$

Banking angle: $\frac{v^2}{rg} = \tan\theta$, $\frac{v^2}{rg} = \frac{\mu + \tan\theta}{1 - \mu\tan\theta}$

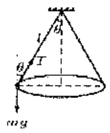
Centripetal force: $F_c = \frac{mv^2}{r}$, $a_c = \frac{v^2}{r}$

Pseudo force: $\vec{F}_{\rm pseudo} = -m\vec{a}_0$, $F_{\rm centrifugal} = -\frac{mv^2}{r}$

Minimum speed to complete vertical circle:

$$v_{\rm min,\,bottom} = \sqrt{5gl}, \quad v_{\rm min,\,top} = \sqrt{gl}$$

Conical pendulum: $T = 2\pi\sqrt{\frac{l\cos\theta}{g}}$



1.4: Work, Power and Energy

Work: $W = \vec{F}\cdot\vec{S} = FS\cos\theta$, $W = \int \vec{F}\cdot d\vec{S}$

Kinetic energy: $K = \frac{1}{2}mv^2 = \frac{p^2}{2m}$

Potential energy: $F = -\partial U/\partial x$ for conservative forces.

$$U_{\rm gravitational} = mgh, \quad U_{\rm spring} = \tfrac{1}{2}kx^2$$

Work done by conservative forces is path independent and depends only on initial and final points: $\oint \vec{F}_{\rm conservative}\cdot d\vec{r} = 0$.

Work-energy theorem: $W = \Delta K$

Mechanical energy: $E = U + K$. Conserved if forces are conservative in nature.

Power $P_{\rm av} = \frac{\Delta W}{\Delta t}$, $P_{\rm inst} = \vec{F}\cdot\vec{v}$

1.6: Rigid Body Dynamics

Angular velocity: $\omega_{\rm av} = \frac{\Delta\theta}{\Delta t}$, $\omega = \frac{d\theta}{dt}$, $\vec{v} = \vec{\omega}\times\vec{r}$

Angular Accel.: $\alpha_{\rm av} = \frac{\Delta\omega}{\Delta t}$, $\alpha = \frac{d\omega}{dt}$, $\vec{a} = \vec{\alpha}\times\vec{r}$

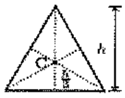
1.5: Centre of Mass and Collision

Centre of mass: $x_{cm} = \frac{\sum x_i m_i}{\sum m_i}$, $x_{cm} = \frac{\int x\,dm}{\int dm}$

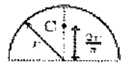
CM of few useful configurations:

1. m_1, m_2 separated by r:

2. Triangle (CM ≡ Centroid) $y_c = \frac{h}{3}$



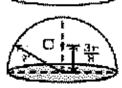
3. Semicircular ring: $y_c = \frac{2r}{\pi}$



4. Semicircular disc: $y_c = \frac{4r}{3\pi}$

5. Hemispherical shell: $y_c = \frac{r}{2}$

6. Solid Hemisphere: $y_c = \frac{3r}{8}$



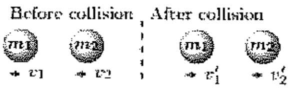
7. Cone: the height of CM from the base is $h/4$ for the solid cone and $h/3$ for the hollow cone.

Motion of the CM: $M = \sum m_i$

$$\vec{v}_{cm} = \frac{\sum m_i \vec{v}_i}{M}, \quad \vec{p}_{cm} = M\vec{v}_{cm}, \quad \vec{a}_{cm} = \frac{\vec{F}_{ext}}{M}$$

Impulse: $\vec{J} = \int \vec{F}\,dt = \Delta\vec{p}$

Collision:



Momentum conservation: $m_1v_1 + m_2v_2 = m_1v_1' + m_2v_2'$
Elastic Collision: $\frac{1}{2}m_1{v_1}^2 + \frac{1}{2}m_2{v_2}^2 = \frac{1}{2}m_1{v_1'}^2 + \frac{1}{2}m_2{v_2'}^2$
Coefficient of restitution:

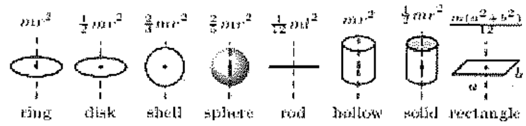
$$e = \frac{-(v_1' - v_2')}{v_1 - v_2} = \begin{cases} 1, & \text{completely elastic} \\ 0, & \text{completely in-elastic} \end{cases}$$

If $v_2 = 0$ and $m_1 \ll m_2$ then $v_1' = -v_1$.
If $v_2 = 0$ and $m_1 \gg m_2$ then $v_2' = 2v_1$.
Elastic collision with $m_1 = m_2$: $v_1' = v_2$ and $v_2' = v_1$.

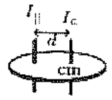
Rotation about an axis with constant α:

$$\omega = \omega_0 + \alpha t, \quad \theta = \omega t + \tfrac{1}{2}\alpha t^2, \quad \omega^2 - {\omega_0}^2 = 2\alpha\theta$$

Moment of Inertia: $I = \sum_i m_i r_i^2$, $I = \int r^2 dm$



Theorem of Parallel Axes: $I_{\parallel} = I_{cm} + md^2$

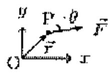


Theorem of Perp. Axes: $I_z = I_x + I_y$

Radius of Gyration: $k = \sqrt{I/m}$

Angular Momentum: $\vec{L} = \vec{r} \times \vec{p}$, $\vec{L} = I\vec{\omega}$

Torque: $\vec{\tau} = \vec{r} \times \vec{F}$, $\vec{\tau} = \frac{d\vec{L}}{dt}$, $\tau = I\alpha$



1.8: Simple Harmonic Motion

Hooke's law: $F = -kx$ (for small elongation x.)

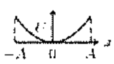
Acceleration: $a = \frac{d^2x}{dt^2} = -\frac{k}{m}x = -\omega^2 x$

Time period: $T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}}$

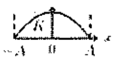
Displacement: $x = A\sin(\omega t + \phi)$

Velocity: $v = A\omega\cos(\omega t + \phi) = \pm\omega\sqrt{A^2 - x^2}$

Potential energy: $U = \frac{1}{2}kx^2$



Kinetic energy $K = \frac{1}{2}mv^2$



Total energy: $E = U + K = \frac{1}{2}m\omega^2 A^2$

Simple pendulum: $T = 2\pi\sqrt{\frac{l}{g}}$

Physical Pendulum: $T = 2\pi\sqrt{\frac{I}{mgl}}$

Torsional Pendulum $T = 2\pi\sqrt{\frac{I}{k}}$

Conservation of $\vec{L}$: $\vec{\tau}_{ext} = 0 \implies \vec{L} = \text{const}.$

Equilibrium condition: $\sum \vec{F} = \vec{0}, \quad \sum \vec{\tau} = \vec{0}$

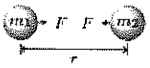
Kinetic Energy: $K_{rot} = \frac{1}{2} I \omega^2$

Dynamics:

$$\vec{\tau}_{cm} = I_{cm}\vec{\alpha}, \qquad \vec{F}_{ext} = m\vec{a}_{cm}, \qquad \vec{p}_{cm} = m\vec{v}_{cm}$$
$$K = \frac{1}{2} m {v_{cm}}^2 + \frac{1}{2} I_{cm}\omega^2, \quad \vec{L} = I_{cm}\vec{\omega} + \vec{r}_{cm} \times m\vec{v}_{cm}$$

1.7: Gravitation

Gravitational force: $F = G\frac{m_1 m_2}{r^2}$



Potential energy: $U = -\frac{GMm}{r}$

Gravitational acceleration: $g = \frac{GM}{R^2}$

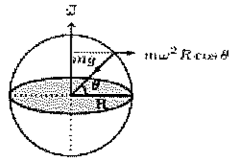
Variation of g with depth: $g_{inside} \approx g\left(1 - \frac{h}{R}\right)$

Variation of g with height: $g_{outside} \approx g\left(1 - \frac{2h}{R}\right)$

Effect of non-spherical earth shape on g:
$g_{at\ pole} > g_{at\ equator}$ ($\because R_e - R_p \approx 21$ km)

Effect of earth rotation on apparent weight:

$$mg'_\theta = mg - m\omega^2 R \cos^2\theta$$



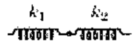
Orbital velocity of satellite: $v_o = \sqrt{\frac{GM}{R}}$

Escape velocity: $v_e = \sqrt{\frac{2GM}{R}}$

Kepler's laws:

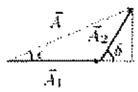
First: Elliptical orbit with sun at one of the focus.
Second: Areal velocity is constant. ($\because \mathrm{d}\vec{L}/\mathrm{d}t = 0$).
Third: $T^2 \propto a^3$. In circular orbit $T^2 = \frac{4\pi^2}{GM}a^3$.

Springs in series: $\frac{1}{k_{eq}} = \frac{1}{k_1} + \frac{1}{k_2}$



Springs in parallel: $k_{eq} = k_1 + k_2$

Superposition of two SHM's:



$$x_1 = A_1 \sin\omega t, \qquad x_2 = A_2 \sin(\omega t + \delta)$$
$$x = x_1 + x_2 = A\sin(\omega t + \epsilon)$$
$$A = \sqrt{{A_1}^2 + {A_2}^2 + 2A_1A_2\cos\delta}$$
$$\tan\epsilon = \frac{A_2 \sin\delta}{A_1 + A_2\cos\delta}$$