BINDURA UNIVERSITY OF SCIENCE EDUCATION
FACULTY OF COMMERCE
DEPARTMENT OF ECONOMICS
BACHELOR OF SCIENCE HONOURS DEGREE IN ECONOMICS
MATHEMATICS FOR ECONOMISTS II EC109 (1)

JUN 2023

DURATION: 3 HOURS   **TOTAL MARKS: 100**

INSTRUCTIONS
1. **This paper carries four questions**
2. **Answer <u>ALL</u> questions.**
3. **All questions carry 25 marks.**
4. **Cellphones are not allowed in the examination room.**

QUESTION 1

a. A consumer has a utility function $X^2 + Y^2$. The price of X is \$1 and Y is \$4. With an income of \$200, how much of the two goods can the consumer purchase to maximise her utility? **[9 marks]**

b. What is the economic interpretation of the Lagrangean multiplier in (a) above? **[2 marks]**

c. Maximize $U(x; z) = x^2 + 3x + 3z^2 - 6z$ subject to $2x + 2z = 32$ **[7 Marks]**

d. Find the extreme values of $f(x; y) = 4x^2 + xy - 3x^2$ **[7 Marks]**

QUESTION 2

a. If the marginal cost of a firm is $C'(Q) = 2e^{0.4Q}$ and the fixed cost is CF=130. Find the total cost function $C'(Q)$. **[5 Marks]**

b. Solve the following integral equations:
   i. $\int 7dx$ **[3 Marks]**
   ii. $\int 12x^{-3}dx$ **[3 Marks]**
   iii. $\int x \sin x \, dx$ **[5 Marks]**
   iv. $\int (6x^3 + 4x^2 - 11)(18x^2 + 4x)dx$ **[5 Marks]**
   v. $\int x \, lnx \, dx$ **[4 Marks]**

QUESTION 3

Solve the following differential equations:

a. $dy/dt + 12y = 10$; y(0) = 0 [3 marks]

b. $dy/dt - 4y = 0$; y(0) = 8 [3 marks]

c. $dy/dt + 5y = 15$ [4 marks]

d. $dy/dt + 2ty = t$; $y(0) = \frac{3}{2}$ [6 marks]

e. $y'' - 6y' + 3y = 10$; $y(0) = 5$ and $y'(0) = 6$ [9 marks]

QUESTION 4

Solve the following difference equations:

i. $y_{t+1} - 4y_t = 6$ when $y_0 = 3$ [4 Marks]

ii. $y_{t+2} + 5y_{t+1} - 2y_t = 8$ when $y_0 = 4$ and $y_1 = 7$ [7 Marks]

iii. $y_{t+2} - 5y_{t+1} + 3y_t = 9$ when $y_0 = 5$ and $y_1 = 9$ [7 Marks]

iv. $y_{t+2} + y_{t+1} - 3y_t = 4$ when $y_0 = 1$ and $y_1 = 4$ [7 Marks]