BINDURA UNIVERSITY OF SCIENCE EDUCATION

MT009: PURE MATHEMATICS 3

JUN 2023

Time : 3 hours

Answer ALL questions in Section A and at most TWO questions in section B.

SECTION A (40 marks)

Candidates may attempt ALL questions being careful to number them A1 to A4.

A1. (a) Find the general solution of the differential equation $\frac{dy}{dx} = (1-y)^2$ expressing y in terms of x. [5]

(b) The complex number z satisfies the equation $|z| = |z+2|$. Show that the real part of z is -1 [3]

A2. Find the general solution of the differential equation $\frac{dy}{dx} + 3x(y^2+4) = 0$ expressing y in terms of x . [5]

A3. (a) Use the trapezium rule to estimate the area under the curve $y = \frac{1}{x}$ from $x = 1$ to $x = 2$. [8]

(b) Use the Simpson's rule to find an approximation for the area under the curve $y = \frac{1}{x}$ between $x = 1$ and $x = 2$. [8]

A4. The lines L_1 and L_2 have equations $r = (3,1,0) + t(1,2,4)$ and $r = (1,-1,1) + s(2,1,-1)$ respectively, where t and s are parameters.

(a) Show that L_1 passes through the point (2,-1,-4) but L_2 does not pass through this point. [4]

(b) Find the acute angle between L_2 and the line joining the points $(1,-1,1)$ and $(2,-1,-4)$ giving your answer correct to the nearest degree. [5]

SECTION B (60 marks)

Candidates may attempt TWO questions being careful to number them B5 to B7.

B5. (a) A curve has an equation $y = (4 - x^2)^{-\frac{1}{2}}$ for $-1 \leq x \leq 1$. The region R is enclosed by $y = (4 - x^2)^{-\frac{1}{2}}$, the x-axis and the line $x = -1$ and $x = 1$.

(i) Find the exact value of the area R. [5]

(ii) Find the exact value of the volume generated when R is rotated through four right angles about the x-axis. [5]

(iii) Show that the volume generated when R is rotated through two right angles about the y-axis is $\pi(4 - 2\sqrt{3})$. [6]

(b) A curve is given by $y^3 + y^2 + y = x^2 - 2x$.

(i) Show that the point $(3, 1)$ is the only point of intersection of the line $x = 3$ and the curve. [5]

(ii) show that the tangent to curve at the point $(-1, 1)$ has equation $2x+3y-1 = 0$. [4]

(iii) Show that at the origin, $\frac{dy}{dx} = -2$ and $\frac{d^2y}{dx^2} = -6$, and give Maclaurin's series for y as far as the term in x^2. [5]

B6. (a) The equation of the line L is $r = (1, 3, 7) + t(2, -1, 5)$. The points A and B have position vectors $(9, 3, 26)$ and $(13, 9, \alpha)$ respectively. The line L intersects the line through A and B.

(i) Find α and the acute angle between line L and AB. [8]

(ii) The point C has position vector $(2, 5, 1)$ and the foot of the perpendicular from C to L is Q. Find the length of CQ . [7]

(b) A curve is given by the parametric equations $x = t^2$, $y = t^3$.

(i) Prove that the equation of the tangent at the point with parameter t is $2y - 3tx = t^3 = 0$. [5]

(ii) This tangent passes through a fixed point (X, Y). Give a brief argument to show that there cannot be more than 3 tangents passing through (X, Y). [3]

(iii) The tangent at the point where $t = 2$ meets the curve again at the point where $t = u$. Find the value of u. [7]

B7. (a) A curve is given by $y = x^5 - 10x$.

(i) Find the coordinates of the turning points on the graph of $y = x^5 - 10x$. [3]

(ii) Show with the aid of a sketch, that the equation $x^5 - 10x = 5$ has three real roots. [5]

(iii) State the two consecutive integers between which the positive root of the equation $x^5 - 10x = 5$ lie. [3]

(iv) Carry out one linear interpolation, starting with these two integers, to obtain an estimate of the positive root. Explain, with reference to a sketch, why this linear interpolation gives an underestimate of the root. [4]

(v) Use the Newton-Raphson method to find the value of the positive root correct to 1 decimal place. [5]

(b) Given the equation $y = \frac{4x}{(x-1)^2}$.

(i) State the equations of the asymptotes of the curve, and use differentiation to find the coordinates of the turning point on the curve. [5]

(ii) Sketch on separate diagrams the graphs of $y = \frac{4x}{(x-1)^2}$ and $y^2 = \frac{4x}{(x-1)^2}$. [5]

END OF QUESTION PAPER