Bindura University of Science Education

Faculty of Science Education

Department of Science and Mathematics Education

Programmes: HBSc Ed (Mathematics)

Course: MT207: Analysis Duration: Three hours

Semester Examinations

JAN 2025

Instructions to candidates

(i) Answer all questions in Section A and two questions from Section B.

(ii) Begin each question on a fresh page.

Section A [40 marks]

Answer all questions from this section being careful to number them **A1** to **A5.**

A1. (a). Define the terms:

(i). infimum of a set $S \subseteq \mathbb{R}$ [2]

(ii). an ordered field. [4]

(b). Find the least upper bound and greatest lower bound for the sets:

(i). $A = \left\{\frac{4+x}{x} | x \geq 1\right\}$, [4]

(ii) $B = \{x | x^2 - x < 6\}$. [3]

A2. Prove that there exists a real number x such that $x^2 = 2$. [8]

A3. A sequence (u_n) has its $n-th$ term given by $(u_n) = \frac{2n-3}{4n+7}$. Prove that (u_n) converges. [7]

A4. (a) Define a Cauchy sequence. [3]

(b). Prove that the sequence $(x_n) = \frac{1}{n}$ is a Cauchy sequence. [4]

A5. Prove that if $\frac{m}{n}$ and $\frac{p}{q}$ are rational numbers with $p \neq 0$, then $\frac{m}{n} + \sqrt{2}\frac{p}{q}$ is an irrational number [5]

Section B:[60 marks]

Answer **two** questions from this section being careful to number them **B6** to **B8.**

B6. (a). Prove that a sequence (a_n) converges to a real number L *iff* for every $\varepsilon > 0$, all but a finite number

of the terms of (a_n) lie in the interval $(L - \varepsilon, L + \varepsilon)$. **[12]**

(b). Prove that if $a, b \in \mathbb{R}$, with $a > 0$, then there is a positive integer n such that $na > b$. **[10]**

(c). Prove that $f(x) = x^2 + 2x - 5$ is uniformly continuous on $[0,2]$. **[8]**

B7. (a). Define a cut in $\mathbb{R}$. **[3]**

(b). Prove that if an ordered paired (A, B) of non-empty subsets of $\mathbb{R}$ form a cut in $\mathbb{R}$,

then there is a unique element ε that satisfies: $a \leq \varepsilon, \forall a \in A$ and $\varepsilon \leq \forall b \in B$. **[12]**

(c). State and prove the nested cells property in $\mathbb{R}$. **[15]**

B8. (a). Let f be a function defined on [a, b] and suppose $f(x)$ is differentiable

at $x_o \in (a, b)$. Prove that f is continuous. **[10]**

(b). Define the following terms:

(i). a partition P of an interval $[a, b]$, **[3]**

(ii). a lower and upper Riemann sums of a function f with respect to the partition P **[4]**

(iii). a refinement Q of P. **[2]**

(c). State Riemann condition for an integrability of a function f. **[4]**

(d). Use the definition of the Riemann integral to evaluate $\int_0^1 x^3 dx$. **[7]**

END OF PAPER