BINDURA UNIVERSITY OF SCIENCE EDUCATION

MT107: CALCULUS

JAN 2025

Time : 3 hours

Answer ALL questions in Section A and at most TWO questions in section B.

SECTION A (40 marks)

Candidates may attempt ALL questions being careful to number them A1 to A4.

A1. (a) Show that the function $f(x) = \frac{3x^2+2}{x+2}$ is injective. [5]

(b) Determine whether $f(x) = \frac{3x^2+2}{x+2}$ is surjective or not. [5]

A2. (a) State Green's Theorem of

(i) Circulation form. [2]

(ii) Flux form. [2]

(b) Evaluate the following limits of functions.

(i) $lim_{x\to 0}\frac{2sin(x) - sin(2x)}{2e^x - 2 - 2x - x^2}$. [5]

(ii) $lim_{x\to\infty}(1 + sin(\frac{x}{3}))^x$. [5]

A3. (a) Find Df and Rf if $f(x) = \frac{1}{\sqrt{3-x}}$. [4]

(b) When do we say a sequence S_n is convergent. [2]

A4. (a) Solve the inequality $\frac{1}{4} < \frac{1}{x+3}$. [4]

(b) Show that $f(x) = |x|$ is not differentiable at $c = 0$. [6]

SECTION B (60 marks)

Candidates may attempt TWO questions being careful to number them B5 to B7.

B5. (a) Evaluate: $\int_0^2 \int_{-1}^2 \int_1^3 (x + y^2 + z^3)\, dx\, dy\, dz.$ [10].

(b) Evaluate the line integral: $\oint_C (3x - y)dx + (x + 5y)dy$, where $C := x^2 + y^2 = 1$. [6]

(c) State the second fundamental theorem of calculus. [2]

(d) Find the area of the region bounded by $f(x) = 4 - 4x^2$ and $g(x) = 1 - x^2$. [8]

(e) State the Mean Value Theorem of differentiation. [4]

B6. (a) Give a detailed sketch of the graph of $y = \frac{x^3}{3x - 2}$. [12]

(b) Let $f(x) = x^2 sin(\frac{1}{x}), x \neq 0$.

(i) Does $f(x)$ have a derivative at $x = 0$? Justify your answer. [4]

(ii) Is $f(x)$ differentiable at $x = 0$, justify your answer. [4]

(c) A box shape X is described by a triple integral: $X = \int_0^3 \int_0^2 \int_0^1 (x+y+z)\, dx\, dy\, dz$. [10]

Evaluate X.

B7. (a) Find the indefinite integral of $\int \frac{x^3 + 2}{x^3 - x} dx$. [7]

(b) (i) State the $\epsilon - N$ definition of the limit of a sequence a_n. [2]

(ii) Hence show that a sequence whose n^{th} term is given by $a_n = (3 - \frac{1}{7n^2})$ converges to 3. [5]

(c) Show that the sequence $U_n = \frac{2n - 7}{3n + 2}$ is monotonic increasing. [6]

(d) Let S be a paraboloid $Z = \frac{x^2 + y^2}{4}$ for $z \leq 4$ oriented with upward normal vector. Use Stokes' Theorem to calculate $\int\int_S curlV.ds$ where $V(x, y, z) = xy^2 i - 4x^2 yj + \frac{z - 1}{x^2 + 2y^2 + 1} k$. [10]

END OF QUESTION PAPER