BINDURA UNIVERSITY OF SCIENCE EDUCATION

FACULTY OF SCIENCE AND ENGINEERING

COMPUTER SCIENCE DEPARTMENT

BSc HONS DEGREE IN COMPUTER SCIENCE/INFORMATION TECHNOLOGY

DISCRETE MATHEMATICS -CS205/CSH105/NWE107/SWE117

2 HOURS 30 MINUTES

MAR 2023

INSTRUCTION TO CANDIDATES
This paper carries **four (4)** questions. Answer **all.**

Marks are indicated in brackets at the end of each question. Total marks are 100

Question 1

a) Assume that you have the flu virus, and you forgot to cover your mouth when two friends came to visit while you were sick in bed. They leave, and the next day they also have the flu. Let's assume that each friend in turn spreads the virus to two of their friends by the same droplet spread the following day. Assuming this pattern continues and each sick person infects 2 other friends, we can represent these events in the following manner shown figure 1:
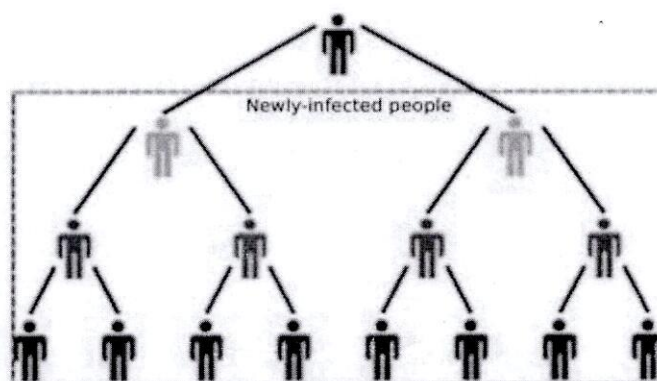

Figure 1: Infection pyramid

i.   Calculate the number of newly-infected people on the tenth day.   **[4]**

ii. On which day will 16 384 people be newly-infected?   **[4]**

b) Find the solution of the recurrence relation

$$a_{n+2} + 8a_{n+1} - 9a_n = 0, \quad n \geq 0$$

satisfying the initial conditions

$$a_0 = 0 \text{ and } a_1 = 10.$$ **[6]**

c) Find the general solution of the recurrence relation

$$a_{n+2} - 5a_{n+1} + 6a_n = 0, \quad n \geq 0.$$ **[6]**

d) Disprove the following statement by finding a counter example:

$\forall$ real numbers a and b, if $a > b$ then $a^2 > b^2$.   **[4]**

Question 2

a) One hundred students were surveyed and asked which email service they use, Yahoo, Gmail or Hotmail. 16 students don't use any of the three, 68 use Yahoo, 28 use Gmail and 35 use Hotmail. 3 students use both Yahoo and Gmail but not Hotmail, 5 use Hotmail and Gmail but not Yahoo and 5 use Yahoo and Hotmail but not Gmail.

i)   Illustrate this information on a Venn Diagram   **[3]**

ii)   How many students use all 3?   **[2]**

iii) How many students use only one of the 3?   **[2]**

b) Let A = {1,2,3,4} and R = {(1,1), (1,2), (1,3), (1,4), (2,2), (2,3), (2,4), (3,2), (3,3), (3,4), (4,3), (4,4)}

For the relation R determine whether it is:

i)   Reflexive.   **[2]**

ii)   Symmetric.   **[2]**

iii) Transitive.   **[4]**

iv)   An equivalence relation   [2]

c) Let $f : \mathbb{R} \rightarrow \mathbb{R}$ and $g : \mathbb{R} \rightarrow \mathbb{R}$ be two functions defined by

$f(x) = x + 2$ and $g(x) = x^2 + x - 6$

Where $x \in \mathbb{R}$

i)   Compute $f(3)$ and $g(4)$.   [2]
ii) Is $g(x)$ injective? Give a reason for your answer.   [3]
iii) What is the range of $f(x)$?   [2]
iv) Is $f(x)$ surjective? Give a reason for your answer.   [3]
v)   Calculate the composite function $g \circ f(x)$.   [3]
vi) Calculate $g^{-1}(x)$ [3]

Question 3
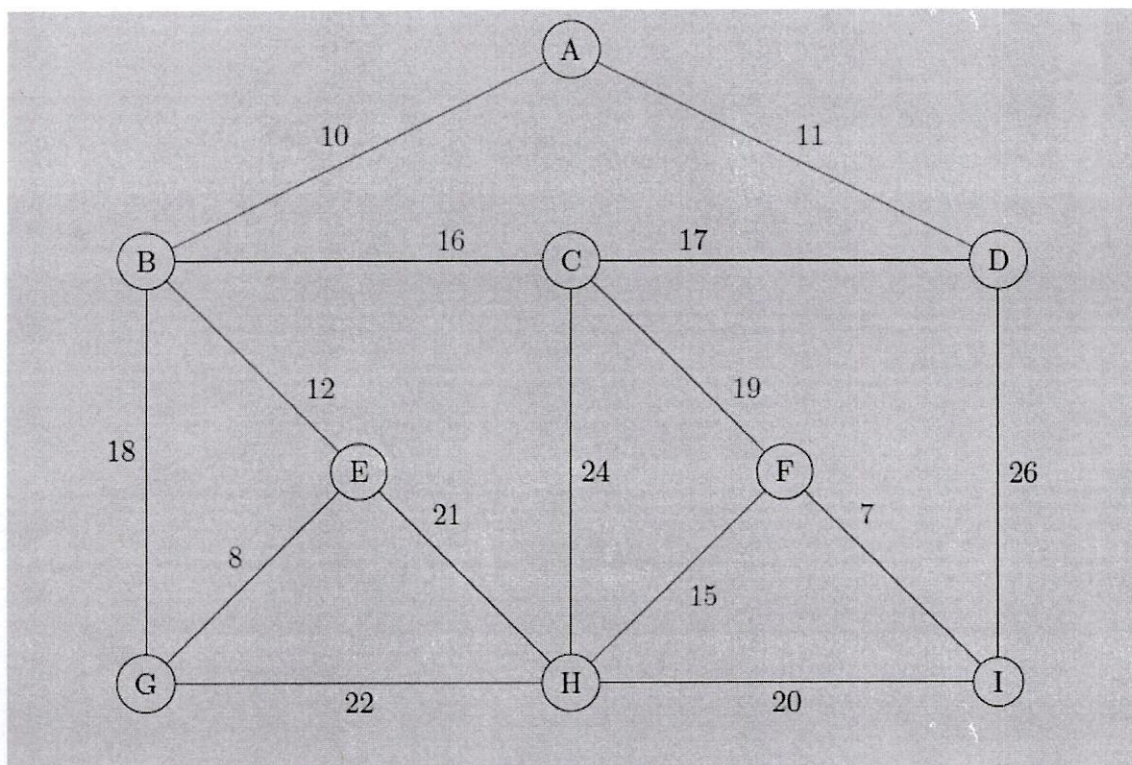
Given the graph in figure 2

Figure 2: Graph

a) Calculate the minimum spanning tree using Prim's algorithm, start with node $v_0 = A$ in the tree. Write the order in which vertices are added to the tree.

[6]

b) You are a travel agent. You have to provide your client the fastest air trip from Ithaca to Seattle. For doing so, you are provided with table 1 showing the flight times from one city to another:

From / To	Ithaca	Detroit	New York	Philadelphia	Denver	Los Angeles	Seattle
Ithaca	-	3	1	2	-	-	-
Detroit	3	-	-	-	7	-	-
New York	1	-	-	2	-	9	-
Philadelphia	2	-	2	-	-	4	-
Denver	-	7	-	-	-	-	5
Los Angeles	-	-	9	4	-	-	2
Seattle	-	-	-	-	5	2	-

Table 1: Flight times

i) Explain why one can quickly look at this table and tell that the corresponding graph is undirected. [2]

ii) Draw the graph for the flights between the cities and label the edges with their weights. [10]

iii) Use Dijkstra's algorithm to find the shortest flight time from Ithaca to Seattle. Write down the nodes in the order you add them to the settled set. For simplicity, ignore any time spent on the ground. [8]

Question 4

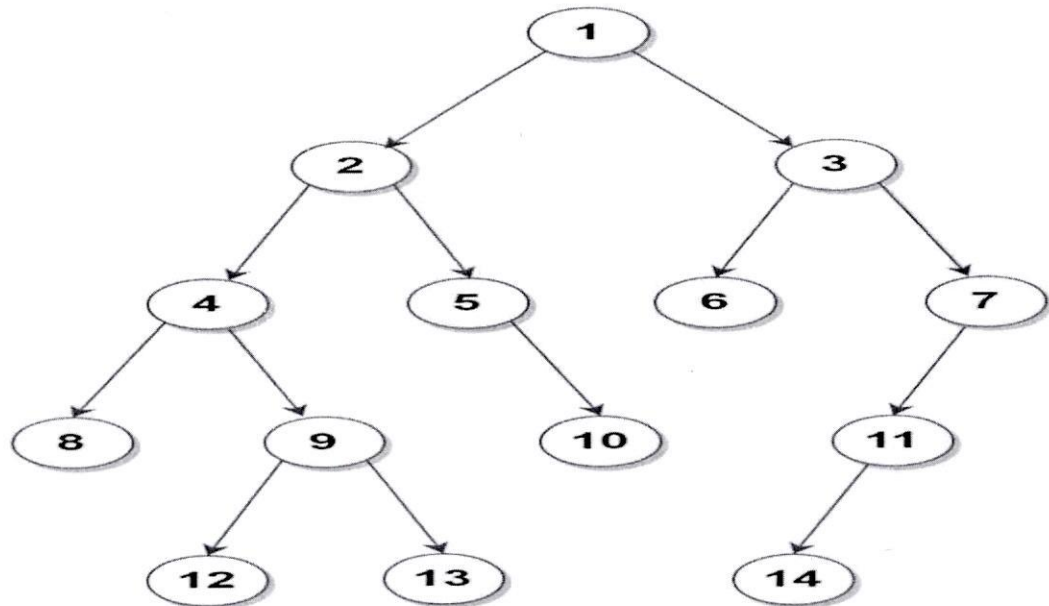a) The expression tree diagram in figure 3 represents an infix expression

Figure 3: expression tree

Write the output if the expression tree is visited using the following methods

    i.   Preorder traversal   **[3]**

    ii.   Inorder traversal   **[3]**

   iii.   Postorder traversal   **[3]**

b) Let P, Q and R and be propositions as follows:

  P: Today is Monday

  Q: Jupiter is a star

  R: Elephants can fly

Write each of the following in terms of p, q, r and logical connectives:

  i)   Today is Monday and Jupiter is not a star   **[2]**

ii)   *Either today is not Monday or Elephant cannot fly*   **[2]**

iii) *Jupiter is a star and today is Monday and Elephants cannot fly*   **[2]**

iv)   *Jupiter is a star and Jupiter is not a star*   **[2]**

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