BINDURA UNIVERSITY OF SCIENCE EDUCATION

FACULTY OF SCIENCE AND ENGINEERING

Department of Engineering and Physics

BACHELOR OF SCIENCE EDUCATION HONOURS DEGREE (PHYSICS)

JUN 2025

PH201

Classical Dynamics

Duration: Three (3) Hours

Answer ALL parts of Section A and any THREE questions from Section B. Section A carries 40 marks and each question of Section B carries 20 marks.

You may not start to read the questions printed on the subsequent pages until instructed to do so by the Invigilator.

SECTION A

Attempt all parts of question 1

1. (a) Give a concise description of the quantity called the action. Hence state the principle of least action.

[10]

(b) Consider a ball thrown through the air whose Lagrangian is

$$\mathcal{L} = \frac{1}{2}m(\dot{x}^2 + \dot{y}^2 + \dot{z}^2) - mgz$$

Find the conserved momenta.

[10]

(c) Find the form of V(r) so that the path of a particle is given by the spiral

$$r = C\theta^k$$

where C and k are constants.

[10]

(d) A string wraps around a uniform cylinder of mass M, which rests on a fixed plane. The string passes up over a massless pulley and is connected to a mass m, as shown in Figure 1.

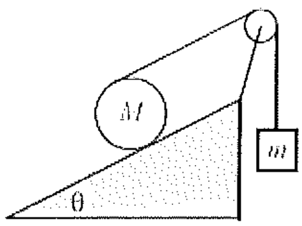

Figure 1

Assume that the cylinder rolls without slipping on the plane, and that the string is parallel to the plane.

(a) What is the acceleration of the mass m? [5]

(b) What is the minimum value of M/m for which the cylinder accelerates down the plane? [5]

SECTION B

Attempt any three (3) questions

2. (a) Write down the Euler-Lagrange equation. Define all the symbols used. [5]

(b) A massless pulley hangs from a fixed support. A massless string of fixed length connecting two masses, m_1 and m_2, hangs over the pulley as in Figure 2.

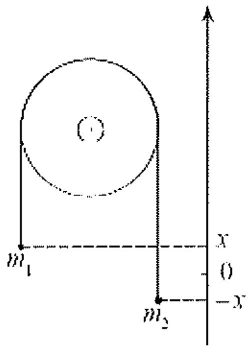

Figure 2

Find the Lagrangian and derive the Euler-Lagrange equation of motion for this dynamical system. [15]

3. A particle moves in a potential

$$V(r) = -\frac{C}{3r^3}$$

(a) Find the maximum value of the effective potential. [10]

(b) Let the particle come in from infinity with speed v_0 and impact parameter b. In terms of C, m, and v_0, what is the largest value of b (call it b_{max}) for which the particle is captured by the potential? In other words, what is the "cross section" for capture, πb_{max}^2, for this potential? [10]

4. A surface of revolution has two given rings as its boundary; see Figure 3.

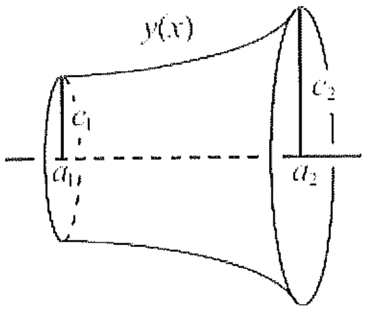

Figure 3

What should the shape of the surface be so that it has the minimum possible area?

[20]

5. For a central field we know that

$$\left(\frac{1}{r^2}\frac{dr}{d\theta}\right)^2 = \frac{2mE}{L^2} - \frac{1}{r^2} - \frac{2mV(r)}{L^2}$$

where symbols have their usual meanings. For finding the equation of the orbit, namely r as a function of θ, subject to a potential of the form $V(r) = \beta r^2$, it is convenient to define the variable $y \equiv 1/r^2$.

(a) Show that

$$\frac{1}{r^2} = \frac{mE}{L^2}(1 + \epsilon cos2\theta); \qquad \epsilon \equiv \sqrt{1 - \frac{2\beta L^2}{mE^2}}$$

where ϵ is the eccentricity of the particle's motion.

[12]

(b) Show that the particle's path is an ellipse.

[8]

6. (a) Show that the moment of inertia I of a thin uniform rod of mass M and length L (axis through centre, perpendicular to the rod) is $ML^2/12$.

[6]

(b) The moment of inertia of an isosceles triangle of mass M, vertex angle 2β, and common-side length L (axis through tip, perpendicular to plane; Figure 4)

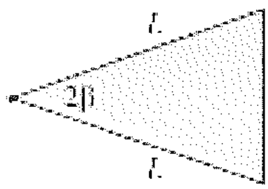

Figure 4

Hence find the moment of inertia of a regular N-gon of mass M and "radius" R (axis through centre, perpendicular to plane). See Figure 5.

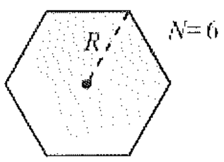

Figure 5

[8]

(d) Using the result of part (c) above and the shorthand notation $(N, I/MR^2)$ list the values for $N = 3;\ 4;\ 6$; and $N \to \infty$.

[6]

Potentially Useful Relations

$$ds^2 = dr^2 + r^2 d\phi^2 + dz^2 \qquad ds^2 = dr^2 + r^2 d\theta^2 + r^2 sin^2\theta d\phi^2$$

$$\frac{d}{dt}\left(\frac{\partial}{\partial \dot{x}}\right) = \frac{\partial L}{\partial x} \qquad \sum_j \dot{q}_j \frac{\partial L}{\partial \dot{q}_j} - L = H \qquad L = T - V$$

$$\dot{p}_k = -\frac{\partial H}{\partial q_k} \qquad \dot{q}_k = \frac{\partial H}{\partial p_k} \qquad P(q,\dot{q}) \equiv \sum_i \frac{\partial L}{\partial \dot{q}_i} K_i(q)$$

$$mr^2\dot{\theta} = L.$$

$$\frac{1}{2}m\dot{r}^2 + \frac{L^2}{2mr^2} + V(r) = E.$$

$$L = mr^2\dot{\theta}$$

$$\frac{dr}{dt} = \pm\sqrt{\frac{2}{m}}\sqrt{E - \frac{L^2}{2mr^2} - V(r)} \qquad E_{total} = \frac{1}{2}m\dot{r}^2 + V(r) + \frac{l^2}{2mr^2}$$

$$\mathcal{L} = \frac{1}{2}m\left(\dot{r}^2 + r^2\dot{\theta}^2\right) - V(r) \qquad V_{eff}(r) = \frac{L^2}{2mr^2} + V(r)$$

$$\frac{1}{r} = \frac{m\alpha}{L^2}(1 + \epsilon cos\theta) \qquad \epsilon \equiv \sqrt{1 + \frac{2EL^2}{m\alpha^2}}$$

$$r_{min} = \frac{L^2}{m\alpha(1+\epsilon)} \qquad r_{max} = \frac{L^2}{m\alpha(1-\epsilon)}$$

$$I_x \equiv \int dm\,(y^2 + z^2) \qquad I_y \equiv \int dm\,(z^2 + x^2) \qquad I_z \equiv \int dm\,(x^2 + y^2)$$

$$I_z = MR^2 + I_z^{CM} \qquad I_z = I_x + I_y$$

$$R_{CM} = \frac{\sum r_i m_i}{M} \quad \Leftrightarrow \quad R_{CM} = \frac{\int dm\, r}{M}$$