Bindura University of Science Education

Faculty of Science Education

Department of Science and Mathematics Education

Programmes: HBSc Ed (Mathematics)

Course: MT207 Analysis Duration: **Three** hours

Semester Examinations

Instructions to candidates

(i) Answer all questions in Section A and two questions from Section B.

(ii) Begin each question on a fresh page.

Section A [40 marks].

Answer all questions from this section being careful to number them **A1** to **A4.**

A1. (a) Prove that $1 + nx \leq (1+x)^n$ for $n \geq 2$. **[7]**

(b). Suppose f and g are continuous on $[a, b]$ and differentiable on (a, b). Prove that if

$f'(x) = g'(x)$ on (a, b) then f and g differ by a constant. **[4]**

(c) Prove that the multiplicative inverse of a non-zero element of a field $\mathbb{R}$ is unique **[5]**

A2. Let $S \subseteq \mathbb{R}$ be non-empty and bounded above. Then for $u \in \mathbb{R}$ to be a least upper bound it is necessary and sufficient that (i) u is upper bound of S,

(ii). for every $\varepsilon > 0, \exists\, x \in S$ such that $u - \varepsilon < x$ **[10]**

A3. Prove that $\lim_{x\to\infty} \frac{\sqrt{3x^2 + 4}}{x} = \sqrt{3}$. **[8]**

A4. Prove that a bounded monotone sequence converges **[6]**

Section B:[60 marks]

Answer **two** questions from this section being careful to number them **B5** to **B7.**

B5. (a) A sequence (a_n) of real numbers is defined by $a_1 = \sqrt{2}$ and $a_{n+1} = \sqrt{2 + a_n}$.

(i). Prove that (a_n) is a bounded monotone increasing sequence. **[8]**

(ii). Hence, determine its limit. [6]

(b) . Prove that a subset of $\mathbb{R}$ is closed if and only if contains its boundary points. **[8]**

(c). Prove that if a function, f is continuous on $[a, b]$ then f is Riemann integrable. **[8]**

B6. (a). Define a cut in $\mathbb{R}$. **[3]**

(b). Prove that if an ordered paired (A, B) of non-empty subsets of $\mathbb{R}$ form a cut in $\mathbb{R}$, then there is a unique element ε that satisfies: $a \leq \varepsilon, \forall a \in A$ and $\varepsilon \leq b \; \forall b \in B$. **[15]**

(c). Prove that a sequence (a_n) converges to a real number L *iff* for every $\varepsilon > 0$, all but a finite number of the terms of (a_n) lie in the interval $(L - \varepsilon, \; L + \varepsilon)$. **[12]**

B7. (a). Let f be a function defined on [a, b] and suppose $f(x)$ is differentiable at $x_o \in (a, b)$ prove that f is continuous at x_0. `**[10]**

(b). Define the following terms:

(i). a partition P of an interval $[a, b]$, **[3]**

(ii). a lower and upper Riemann sum of a function f with respect to the partition P **[3]**

(c). State Riemann condition for an integrability of a function f. **[4]**

(d).State and prove the intermediate value theorem (IVT). **[8]**

END OF PAPER