BINDURA UNIVERSITY OF SCIENCE EDUCATION

DEPARTMENT OF STATISTICS AND MATHEMATICS

AMT102/ SFM 112: LINEAR ALGEBRA AND FINANCIAL MODELING
/LINEAR ALGEBRA 1

Time: 3 hours

SECTION A

Candidates may attempt at most **TWO** questions being careful to number
1 to 4

1. (a) Define the following terms:

   i. linear differential equation [2]

   ii. integrating factor. [2]

   iii. wronskian. [2]

   iv. the order of a differential equation. [2]

   (b) Prove that
   $$(AB)^{-1} = B^{-1}A^{-1}$$

   [4]

   (c) Briefly describe principal component analysis (PCA) as applied to
   finance and risk management. [11]

2. What do you understand by the term Bernoulli equation? [3]

   Hence or otherwise, solve the equation

   $$\frac{dy}{dx} + y = xy^4$$

   [5]

3. Determine the definiteness of the following matrix:

   $$\begin{bmatrix} 1 & 1 & 2 \\ 1 & 5 & 4 \\ -2 & -4 & 6 \end{bmatrix}$$

   [5]

4. A substance cools from 96 degrees Celsius to 60 degrees Celsius in 15 seconds. Find the temperature of the substance after 40 seconds assuming the ambient temperature is 20 degress Celsius.

   Hint: Use Newton Law of cooling.                                      [4]

## SECTION B

Candidates may attempt at most **TWO** questions being careful to number 5 to 7

5. (a) What do you understand by the following terms in matrix algebra?
   i. LU decomposition. [2]
   ii. cholesky decomposition. [2]
   iii. normalised eignevectors. [2]
   iv. singular matrix. [2]

   (b) Perform an LU decomposition of the following matrix:

$$\begin{bmatrix} 1 & 2 & 0 & 1 \\ 2 & 2 & 2 & -3 \\ 0 & 1 & 3 & 8 \\ 1 & -1 & 8 & -2 \end{bmatrix}$$

[5]

   (c) Evaluate the determinant of the matrix above. [7]

   (d) Use Cramer's Rule to solve the following system of equations:

$$7x - y - z = 0$$
$$10x - 2y + z = 8$$
$$6x + 3y - 2z = 7$$

[10]

6. (a) Evaluate the eigenvalues and eigenvectors of the matrix:

$$\begin{bmatrix} 5 & 2 & 0 \\ 2 & 5 & 0 \\ -3 & 4 & 6 \end{bmatrix}$$

[8]

   (b) Solve the following system of equations using the Gaussian elimination method:

$$x + y + z = 2$$
$$10x + 2y + 3z = 5$$
$$2x + 3y + 4z = 11$$

[6]

(c) Use the method of variation of parameters to solve the equation:

$$\frac{d^2y}{dx^2} + y = secx$$

[8]

(d) A certain radioactive material is known to decay at a rate proportional to the amount present and 0.1 percent of the original mass has decayed in one week. Find an expression for the mass at time $t$. [8]

7. (a) Define the terms ordinary and regular singular points in connection with a generic second order linear differential equation. [4]

(b) State Frobenius Theorem for solution of differential equations about a regular singular point. [4]

(c) Show that $x = 0$ is a regular singular point of the equation

$$2x\frac{d^2y}{dx^2} + (1 + x)\frac{dy}{dx} = 0$$

and obtain a series solution about $x = 0$. [22]

END OF PAPER

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