BINDURA UNIVERSITY OF SCIENCE EDUCATION

FACULTY OF SCIENCE EDUCATION

Diploma in Science Education Part 1.1

JUN 2023

DM008: Numerical Methods Duration 3 hours

Semester Examinations

INSTRUCTIONS

Answer all questions in Section A and any **two** questions from Section B

Section A: (40 marks)

A1. (i) Use Simpson's rule to estimate the value of $\int_0^{0.8} \sqrt{(1+x^2)}\, dx$ using ordinates at intervals of 0.2. **[8]**

(ii) Suggest two reasons why numerical solutions are important. **[2]**

A2. (a) Variables x and y are connected by the relation a law of the form $y = kx^n$, where k and n are constant integers.

(i) Transform the relation $y = kx^n$ to linear form. **[3]**

Approximate values of y corresponding to the given values of x are given in the following table.

x	1.34	3.58	7.6	12.1	14.8
y	208.0	10.9	1.14	0.283	0.155

Find, by plotting a suitable graph the values of k and n. **[7]**

A3. (a) Use graphical method to find an approximate solution to the equation $e^x = 2x + 1$. **[4]**

(b) Using the solution in (a) as an initial approximation, apply two iterations of the Newton-Raphson iteration to find a better approximation to the root to 3 significant figures. **[6]**

A4. (a) The chord AB of a circle divides the circle into two portions whose areas are in the ratio 3: 1. If AB makes an angle θ with the diameter passing through A, show that θ satisfies the equation: $\sin 2\theta = \frac{\pi}{2} - 2\,\theta$. **[5]**

(b) Solve the equation $\sin 2\theta = \frac{\pi}{2} - 2\,\theta$ by graphical means. **[5]**

Section B [60 marks]

Answer **two** questions from this section being careful to number them **B5** to **B7.**

B5. (a) If x_1 is the first approximation of the root of the equation $f(x) = 0$ and x_2 is the second approximation of the root:

(i) State the algebraic connection between x_1 and x_2. **[2]**

(ii) Hence, show that $x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$. **[4]**

(iii) Apply the Newton-Raphson method to find the root of the equation $x + \frac{4}{x^2} - 1$, with $x_0 = -1$, giving your answer to 3 significant figures. **[7]**

(b) (i) Show that if x is a fixed point of the iteratin then, $x_{n+1} = \sqrt{3x_n + 2},$ then x satisfies the equation: $x^2 - 3x - 2 = 0$.

(ii) Perform 4 iterations of for $x_{n+1} = \sqrt{3x_n + 2},$ using $x_0 = 1$ to obtain x_4. **[6]**

(c). (i) Use linear interpolation to find the root of the equation $e^x = 3x + 1$ to 3 decimal places. **[5]**

(ii) It is known that x and y are relaed by the law $ae^y = x^2 - bx$. Explain how you would reduce the relation to the form $Y = mX + c$. **[6]**

B 6. (a) (i) Use Simpson's rule with 7 ordinates to find an estimate for the value of $\int_0^{0.6} xe^x\,dx$. **[7]**

(ii) Find by analytic means the exact value of $\int_0^{0.6} xe^x\,dx$. **[5]**

(iii) Hence, determine the absolute error that resulted fro m use of Simpson's rule. **[4]**

(b) (i) If x is small enough that terms involving x^5 and higher powers of x can be ignored, use the Binomial Theorem to show that $\frac{1}{\sqrt{1+x^2}} = 1 - \frac{1}{2}x^2 + \frac{3}{8}x^4$. **[6]**

(ii) Hence, show that the approximate value of the integral $\int_0^{0.1} \frac{1}{\sqrt{1+x^2}}\,dx = 0.0998$. **[8]**

B7. (a) (i). If $I = \int_a^b f(x)dx$ then prove that $I = \frac{b-a}{n}[y_0 + y_n + 2(y_1 + y_2 + \cdots + y_{n-1})]$, where n denotes the number of strips. **[6]**

(ii). Estimate the value of $\int_1^e lnxdx$ using the trapezium rule with 5 ordinates to 4 decimal places. **[6]**

(iii). Hence, determine the relative error in the trapezium rule. **[3]**

(b) The sequence given by the iterative formula: $x_{n+1} = 2(1 + e^{-x_n})$ with $x_0 = 0$, converges to the root, α. Find, an estimate for α to 3 decimal places and state the equation which α is a root. **[7]**

(c) (i) Verify that the equation $x^3 - 7x + 3 = 0$ has a root between 2 and 3. **[3]**

(ii). Use the interval bisection method to determine an approximation of the root in (i) to 3 decmal places. **[5]**

END OF PAPER