BINDURA UNIVERSITY OF SCIENCE EDUCATION

SFM413: FINANCIAL DERIVATIVES

OCT 2024

Time : 3 hours

Answer ALL questions in Section A and at most TWO questions in section B.

SECTION A (40 marks)

Candidates may attempt ALL questions being careful to number them A1 to A6.

A1. When can holders of put options exercise or fail to exercise their options. [4]

A2. Distinguish between the following terms.

(a) European and American options [2]

(b) Hedging and Speculation [3]

(c) Forwards and Futures [3]

A3. Denote the European call option price in the Black- Scholes model by

$$C^E = SN(d_1) - Xe^{-rT}N(d_2)$$

where

$$d_1 = \frac{ln\frac{S}{X} + (r + \frac{1}{2}\sigma^2)T}{\sigma\sqrt{T}}$$

Prove that the vega of an option is given by

$V = \frac{s\sqrt{T}}{\sqrt{2\pi}}e^{-\frac{d_1^2}{2}}$. [6]

A4. Let $S_0 = \$60$, $r = 5\%$, $u = 0.3$ and $d = -0.1$. Find the prices of European call and put options with strike price $K = \$60$ to be exercised after $N = 3$ steps. Assuming periodic compounding. [8]

A5. Explain the following assumptions for building a mathematical model of a market of financial securities.

(a) Liquidity. [2]

(b) Positivity. [2]

(c) Short selling. [2]

A6. Compute the value of an American call option expiring at time 3 with strike price $K = \$62$ on a stock with initial price $S_0 = \$60$ in a binomial tree model with $u = 0.1$, $d = -0.05$ and $r = 0.03$. [8]

SECTION B (60 marks)

Candidates may attempt TWO questions being careful to number them B7 to B9.

B7. (a) The Black Scholes formula has been used to price European and American options among others.

(i) What factors affect the Black Scholes formula. [5]

(ii) State the Black Scholes formula for European put options, clearly giving meaning of each parameter. [6]

(iii) State the assumptions made in order to apply the Black Scholes formula. [5]

(b) Consider a European call option with 20 days to expiration. The strike price is \$105 and the price of the stock is \$100 and the stock has a daily volatility of \$0.02. Assume an interest rate of 0.01 (1% annual). Calculate the price of a European put option. [9]

(c) State the put- call parity theorem clearly explaining parameters. [2]

(d) Consider a European call option on a non dividend paying stock when the stock price is \$51, strike price is \$50, the time to maturity is 6 months and the risk free interest rate is 12% per annum. Find the lower bound for the option. [3]

B8. (a) State and prove the Cox- Ross Rubinsten formula. [15]

(b) A stock is currently \$100 over each of the next 4 six months period. It is expected to go up by $u = 0.2$ or down by $d = -0.2$ The risk free interest rate is 10% per annum. What is the price of the European call option with a strike price of \$100? Appply the CRR formula assuming periodic compounding. [10]

(c) Prove that the future value V_t increases if any one of the parameters m, t, r or P increases while others remain constant. . [5]

B9. (a) When is a stochatic process X_t with $E[X_t] < \infty$ called

(i) a martingale. [2]

(ii) a sub- martingale. [2]

(iii) a super martingale. [2]

(b)

(c) Show that if B_t,$t \geq 0$ is a standard Brownian motion then

(i) B_t is an f_t - martingale. [7]

(ii) $x_t = e^{\sigma B_t - \frac{\sigma^2 t}{2}}$ is an f_t martingale. [7]

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(d) Prove that if the interest rate is constant, the futures price is $f(0,T) = F(0,T)$ [10]

END OF QUESTION PAPER