BINDURA UNIVERSITY OF SCIENCE EDUCATION

HONOURS DEGREE IN SCIENCE EDUCATION (HBScED) JAN 2024

MT304: Vector Calculus

Time: 3 hours

Candidates may attempts ALL questions in Section A and at most TWO questions in Section B. Each question should start on fresh page.

SECTION A (40 marks)

Candidate may attempt ALL questions being careful to number them A1 to A5

A1. Distinguish between

(a) vector and scalar and give example of each. [3,3]

(b) scalar product and vector product. [2, 2]

A2. Suppose $\bar{V} = (3, 3, 2)$, $\bar{W} = (2, 4, 4)$ and $\bar{X} = (1, 0, 2)$ then find the volume of the parallelepiped made of VWX. [4]

A3. A space curve is described by $\bar{r}(t) = 2\cos 3t\, i + 3\sin 3tj - 3k$. Find:

(i) the unit tangent to the curve, [3]

(ii) the acceleration to the particle. [3]

Find the principle normal vector for helix given by

$\bar{r}(t) = -3\cos 3t\, i + 2\sin 3t\, j + tk.$ [6]

A4. (a) Verify the divergence theorem for $\vec{A} = 4xzi - y^2j + yzk$, where S is the surface area bounded by $x = 0, x = 2, y = 0, y = 2, z = 0, z = 2.$ [5]

(b) Show that the vector field

$\vec{V} = (2x\sin 2y + \cos 3y)i - (\sin x - 2z)j - 2z\sin 2yk$ is solenoidal. [4]

A5. Given that $\vec{E}$(x, y)= $x^2i + y^2j$, *show that* $\overrightarrow{E_x} \cdot \overrightarrow{E_y} = 0$. [5]

SECTION B (60 marks)

Candidates may attempt TWO questions being careful to number them B6 to B8

B6. (a) Let $\bar{F}(x, y, z) = \bar{F}_1(x, y, z)i + \bar{F}_2(x, y, z)j + \bar{F}_3(x, y, z)k$. Show that

$$\bar{F}_z(x, y, z) = \frac{\partial \bar{F}_1}{\partial z} i + \frac{\partial \bar{F}_2}{\partial z} j + \frac{\partial \bar{F}_3}{\partial z} k.$$ [6]

(b) Suppose $\bar{M}(x, y) = e^{xy}i + (3x^2 - y)j + y^2\cos xk$. Calculate:

(i) $\overline{M_x}$, [2]