BINDURA UNIVERSITY OF SCIENCE EDUCATION

MT111: CALCULUS 2

Time : 3 hours

Candidates may attempt ALL questions in Section A and at most TWO questions in Section B. Each question should start on a fresh page.

SECTION A (40 marks)

Candidates may attempt ALL questions being careful to number them A1 to A5.

A1. (a) Define a real valued function of two variables $Z = f(x, y)$. [3]

(b) Determine and sketch the domain of the following function:

$f(x, y, z) = \dfrac{1}{\sqrt{x^2 + y^2 + z^2 - 16}}$. [6]

A2. Discuss continuity of the function given by $f(x, y) = \dfrac{x^3y^2}{1 - xy}$. [5]

A3. (a) Show that the function $z = \sin(x - ct)$ satisfies the wave equation

$$\frac{\partial^2 z}{\partial t^2} = c^2 \frac{\partial^2 z}{\partial x^2}.$$

[8]

(b) Given that $x^2y + y^2z + z^2x = 0$, find z_x. [4]

A4. Find the equation of the tangent plane and normal line to $x^2 + y^2 + z^2 = 30$ at the point $(1, -2, 5)$. [3,3]

A5. (a) Given that $\int\int_R xe^{xy} dA = \int_0^1 \int_{-1}^2 xe^{xy} dx dy$, reverse the order of integration and evaluate the iterated integral. [4]

(b) Evaluate

$$\int_0^{2\Pi}\int_1^2\int_0^{r\cos\theta+2}(r\sin\theta)r\,dz\,dr\,d\theta.$$

[4]

SECTION B (60 marks)

Candidates may attempt TWO questions being careful to number them B6 to B8.

B6. (a) Find the directional derivative of the function $f(x,y) = x^2 + y^2$ in the direction of the vector $\overrightarrow{u} = \cos\theta i + \sin\theta j$ when $\theta = \frac{\pi}{4}$ at the point $(2,3)$. [4]

(b) (i) Find the equation of the tangent plane and the normal line to the paraboloid $z = \dfrac{x^2+4y^2}{10}$ at the point $(1,-1,1)$. [5]

(ii) Given the function $z = 2 - x^2 - y^2$ find the total differential of the function at $(-1,2)$. [3]

(iii) Given $w = \sqrt{x^2+y^2}$ and $x = \sin t$, $y = e^t$, find w_t. [4]

(c) Given that $3x^2z - x^2y^2 + 2z^3 - 5 = 0$, find z_x and z_y. [5]

(d) Find the critical points of the function $h(x,y) = x^2 - y^2 - 2x - 4y - 4$ and classify them. [4]

(e) Find the linearisation of the function $f(x,y) = e^x \sin xy$ at the point $P(0, \frac{\pi}{4})$ and estimate the upper bound error of approximation of $f(x,y) \approx L(x,y)$ over the rectangular region $R : |x| \leq 0.1$ and $|y - \frac{\pi}{4}| \leq 0.1$. [3,2]

B7. (a) State the Fubuni's Theorem for evaluating double integrals. [4]

(b) Using the Fubuni's Theorem, evaluate $\int\int_R f(x,y)dA$ for which $f(x,y) = 1 - 6x^2y$ and $R : 0 \leq x \leq 2,\ -1 \leq y \leq 1$. [6]

(c) Evaluate, using an appropriate change of variable $\int_0^1\int_0^{\sqrt{1-x^2}}(x^2+y^2)dydx$. [7]

(d) (i) State the Stokes Theorem. [3]

(ii) Use the Stokes Theorem to find the work done by a particle moving in a vector field given by:

$$F(x,y) = 2zi + xj + y^2k$$

where S is the surface of the paraboloid $z = 4 - x^2 - y^2$ and C is the trace of S in the xy-plane. [10]

B8. (a) Use the method of Lagrange Multipliers to find the constrained relative maximum of the function $f(x,y) = 1 - x^2 - y^2$ subject to $x + y = 2$. [5]

(b) Sketch the quadratic surface $Z = x^2 - y^2$. [5]

(c) Evaluate the iterated integral $\int_0^2 \int_0^x \int_0^{x+y} dzdydx$. [6]

(d) Given that the vector function $F(x, y) = 3e^{x^2} i - ln(\cos y) j$, find

(i) the divergence of F at the point $(0, \frac{\pi}{6})$, [2]

(ii) the curl of F, and [2]

(iii) verify the Green's theorem for the field $F(x, y) = (x - y)i + xj$ and the region R bounded by the unit circle $C : r = (\cos t)i + (\sin t)j$, $0 \leq t \leq 2\pi$. [10]

END OF QUESTION PAPER