BINDURA UNIVERSITY OF SCIENCE EDUCATION   SFM223

HBSc. in Statistics and Financial Mathematics

NUMERICAL METHODS IN FINANCE

Time : 3 hours   OCT 2024

Candidates may attempt ALL questions in Section A and at most TWO questions in Section B. Each question should start on a fresh page.

SECTION A (40 marks)

Candidates may attempt ALL questions being careful to number them A1 to A6.

A1. Distinguish the following;

    (a) numerical instability and ill conditioned problem,   [2]

    (b) truncation errors and rounding errors,   [2]

    (c) underflow situation and of overflow situation.   [2]

    (d) relative error and absolute error.   [2]

A2. Define the following;

    (a) double precision,   [2]

    (b) floating point,   [2]

    (c) chopping.   [1]

A3. (a) Determine a formula which relates the number of iterations, n, required by the bisection method to converge to within an absolute error tolerance of ϵ, starting from the initial interval (a, b).   [5]

    (b) Find n for the interval $(0, 1)$, with $\epsilon = 0.1$ [2]

A4. Show that when Newton's method is applied to the equation $x^2 - a = 0$, the resulting iteration function is

$$g(x) = \frac{1}{2}(x + \frac{a}{x})$$

       [6]

A5. Solve the using the normal equations approach are given. $Ax = b$ where $A = \begin{pmatrix} 1 & 1 \\ 1 & 2 \\ 1 & 3 \\ 1 & 4 \end{pmatrix}$

and $b = \begin{pmatrix} 2 \\ 1 \\ 1 \\ 1 \end{pmatrix}$. [6]

A6. Find the appropriate Lagrange interpolating polynomial using the table: [8]

x	0.0	0.5	1.0	1.5
f(x)	1	2	3	4

SECTION B (60 marks)

Candidates may attempt TWO questions being careful to number them B7 to B9.

B7. (a) State any two assumptions made on Jacobi Method. [2]

(b) Distinguish the Jacobi and the Gaul Seidel Methods of linear approximation. [6]

(c) Apply the Jacobi method to solve

$$5x_1 - 2x_2 + 3x_3 = -1$$

,

$$-3x_1 + 9x_2 + x_3 = 2$$

,

$$2x_1 - x_2 - 7x_2 = 3$$

Continue iterations until two successive approximations are identical when rounded to three significant digits. [20]

(d) Under what condition does both the Jacobi and Gauss-Seidel methods converge to the unique solution? [2]

B8. (a) Why is a polynomial function a preferred choice for an interpolating functions relative to other choices such as a trigonometric and exponential functions? [3]

(b) The data given below is for distance covered by a body at a specified period.

x	0	1	2	4	6
f(x)	1	9	23	93	259

Construct the Newton divided difference table for the data and find $f_n(4.2)$. [15]

(c) Solve the system of non-linear algebraic equations

$$x_2 - e^{-x_1} + 1 = 0$$

$$x_1^3 - x_2 = 0$$

using Newton's method (do two iterations). [12]

B9. (a) Define the Natural Cubic Spline function. [5]

(b) Find the LU decomposition of $A = \begin{bmatrix} 3 & 1 & 6 \\ -6 & 0 & -18 \\ 0 & 8 & -17 \end{bmatrix}$. [10]

(c) Use 4th order Runge Kutta methods to solve

$$\frac{dy}{dx} = -2y + x + 4$$

, $y(0) = 1$ and $h = 0.2$. [15]

END OF QUESTION PAPER