BINDURA UNIVERSITY OF SCIENCE EDUCATION MT102

BACHELOR OF SCIENCE EDUCATION

LINEAR MATHEMATICS 1

JAN 2025

Time: 3 Hours

Candidates may attempt ALL questions in Section A and at most TWO questions in Section B.

Each question should start on a fresh page.

SECTION A (40 marks)

Candidates may attempt ALL questions being careful to number them A1 to A6.

A1. (a) Find the parametric equation of a line in space which passes through the points P(2, 4, −1) and Q(5,0, 7). [3]

(b) Find the co-ordinates of the point where the line intersects the xy plane. [4]

A2. Plot the point corresponding to $z = -3 + 4i$ in the complex plane, and write an expression for z in polar form. [4]

A3. Given that **u** = 3**i**−2**j**+6**k** and **v** = 2**i**+3**j** − 4**k**, find **u** × **v** and a unit vector perpendicular to the plane containing the vectors **u** and **v**. [4]

A4. (a) Let $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 1 & 0 & 6 \end{bmatrix}$. Find the adjoint of the matrix A and hence or otherwise find A^{-1}, the inverse of matrix A. [7]

(b) Outline four transpositional properties. [4]

A5. Find the solution of the following system of linear equations using Gauss elimination method

$$x_1 - x_2 - 2x_3 = 0$$
$$5x_1 - 2x_2 + x_3 = 1$$
$$2x_1 - x_2 - 3x_3 = -2.$$
[6]

A6. (i) Prove that if A is an nxn matrix and it is invertible, then the $det\ A \neq 0$. [3]

(ii) Given that A=$\begin{bmatrix} 2 & -5 & -1 & 4 \\ -3 & 2 & 0 & 3 \\ 1 & 3 & 2 & 1 \\ 4 & 0 & 6 & -2 \end{bmatrix}$, find the determinant of A. [5]

SECTION B (60 marks)

Candidates may attempt TWO questions being careful to number them B7 to B9.

B7. (a) Outline two advantages and two disadvantages of using the Crammers rule when solving systems of equations. [5]

(b) Find the solution to the linear system of equations,

$$2x_1 + 3x_2 - x_3 = 4$$
$$x_1 - 2x_2 + x_3 = 6$$
$$x_1 - 12x_2 + 5x_3 = 10$$

using Crammers rule. [7]

(c) Given $A = \begin{bmatrix} -1 & 0 & -2 \\ 1 & 1 & 3 \end{bmatrix}$. Find A^tA. [3]

(d) Find x such that $\det A = 0$ if $A = \begin{bmatrix} 1 & x & x \\ -x & -2 & x \\ x & x & 3 \end{bmatrix}$. [5]

(e) Solve the following system of simultaneous equations using Gauss elimination.

$$x - 2y + z - w = 2$$
$$-x + 4y - 2z + 3w = -4$$
$$2x + z + 3w = 9$$
$$-2x + 6y + 5w = 2.$$ [10]

B8. (a) Prove that if $\boldsymbol{a} \times \boldsymbol{b} = 0$ and $\mathbf{a} \neq 0$ and $\mathbf{b} \neq 0$ then $\boldsymbol{a}$ is parallel to $\boldsymbol{b}$. [5]

(b) Find the area of a triangle with the following vertices P(2,1, 0), Q(3, -1, 1) and R(1, 0, 2). [5]

(c) Find the equation of a plane containing the points: M(1, 2, 1), Q(-1, 1, 3) and R(-2, -2, -2). [8]

(d) Find the angle between the planes given by $2x - 3y + 2z = 5$ and $x + 2y - z = 4$. Find also the cartesian equations for their line of intersection. [7]

(e)Find the cartesian equation of a line in space which passes through the points P(1; 0; −1) and Q(3; −2; −3). [5]

B9. (a) Given $z_1 = 3 - 5i$ and $z_2 = 2 - 2i$. Calculate

(i) $z_1\overline{z_2}$ (ii) $\frac{z_2}{z_1} - z_2$ [4]

(b) Solve the equation $2z^5 - 14z^4 + 34z^3 - 34z^2 + 32z - 20 = 0$ given that $3 - i$ is a root of the equation. [8]

(c) Show that $cos^4\theta = \frac{1}{8}cos4\theta + \frac{1}{2}cos2\theta + \frac{3}{8}$. [5]

(d) Prove the following identity using Demoivre's theorem

$$cos5\theta = 16cos^5\theta - 20cos^3\theta + 5cos\theta.$$

[6]

(e)(i) Find the real number of the complex number

$$\frac{(1+i)^2}{3+3i}.$$

[2]

(ii) Verify by calculation that the values of $\frac{z}{z^2+1}$ for $z = x + yi$ and $z = x - yi$ are complex conjugate. [5]

END OF QUESTION PAPER