JAN 2025

BINDURA UNIVERSITY OF SCIENCE EDUCATION

Faculty of Science and Engineering

Department of Engineering and Physics

BACHELOR OF SCIENCE HONOURS DEGREE

Physics Education

Environmental Physics and Energy Sources

PH104/HPH122

Optics and Modern Physics

Duration: Three (3) Hours

Answer ALL parts of Section A and any THREE questions from Section B.

Section A carries 40 marks and each question of Section B carries 20 marks.

Clearly show ALL working

You may not start to read the questions printed on the subsequent pages until instructed to do so by the Invigilator.

Some Fundamental Constants*

QUANTITY	SYMBOL	VALUE
Atomic mass unit	u	1.6605×10^{-27} kg 931.49 MeV/c^2
Avogadro's number	N_A	6.022×10^{23} particles/mole
Bohr magneton	$\mu_B = \frac{e\hbar}{2m_e}$	9.274×10^{-24} J/T 5.788×10^{-5} eV/T
Bohr radius	$a_0 = \frac{\hbar^2}{m_e e^2 k}$	0.5292×10^{-10} m
Boltzmann's constant	k_B	1.381×10^{-23} J/K 8.617×10^{-5} eV/K
Coulomb constant	$k = 1/(4\pi\epsilon_0)$	8.988×10^9 N · m^2/C^2
Electron charge	e	1.602×10^{-19} C
Electron mass	m_e	9.109×10^{-31} kg 5.486×10^{-4} u 0.5110 MeV/c^2
Gravitational constant	G	6.673×10^{-11} N · m^2/kg^2
Hydrogen ground state energy	$E_0 = -\frac{m_e e^4 k^2}{2\hbar^2}$	-13.61 eV
Neutron mass	m_n	1.675×10^{-27} kg 1.009 u 939.6 MeV/c^2
Nuclear magneton	$\mu_n = \frac{e\hbar}{2m_p}$	5.051×10^{-27} J/T 3.152×10^{-8} eV/T
Permeability of free space	μ_0	$4\pi \times 10^{-7}$ N/A^2
Permittivity of free space	ϵ_0	8.854×10^{-12} C^2/N · m^2
Planck's constant	h	6.626×10^{-34} J · s 4.136×10^{-15} eV · s
	$\hbar = h/2\pi$	1.055×10^{-34} J · s 6.582×10^{-16} eV · s
Proton mass	m_p	1.673×10^{-27} kg 1.007 u 938.3 MeV/c^2
Rydberg constant	$R = \frac{m_e k^2 e^4}{4\pi c\hbar^3}$	1.097×10^7 m^{-1}
Speed of light in vacuum	c	2.998×10^8 m/s
Stefan-Boltzmann constant	σ	5.6705×10^{-8} W/m^{2}K^4

Useful Conversions and Combinations

$$1\ \text{eV} = 1.602 \times 10^{-19}\ \text{J}$$
$$1\ \text{cal} = 4.184\ \text{J}$$
$$1\ \text{u} = 931.5\ \text{MeV}/c^2$$
$$1\ \text{MeV}/c^2 = 1.073 \times 10^{-3}\ \text{u} = 1.783 \times 10^{-30}\ \text{kg}$$
$$1\ \text{Å} = 10^{-10}\ \text{m} = 0.1\ \text{nm}$$
$$1\ \text{fm} = 10^{-15}\ \text{m}$$
$$1\ \text{in} = 2.540\ \text{cm}$$
$$1\ \text{mi} = 1609\ \text{m}$$
$$hc = 1.240 \times 10^3\ \text{eV}\cdot\text{nm} = 1.986 \times 10^{-25}\ \text{J}\cdot\text{m}$$
$$\hbar c = 1.973 \times 10^2\ \text{eV}\cdot\text{nm} = 3.162 \times 10^{-26}\ \text{J}\cdot\text{m}$$
$$k_B T = 0.02525\ \text{eV at } T = 300\ \text{K}$$
$$ke^2 = e^2/4\pi\epsilon_0 = 1.440\ \text{eV}\cdot\text{nm}$$
$$1\ \text{barn} = 10^{-28}\ \text{m}^2$$
$$1\ \text{curie} = 3.7 \times 10^{10}\ \text{decays/s}$$
$$1\ \text{MeV}/c = 5.344 \times 10^{-22}\ \text{kg}\cdot\text{m/s}$$

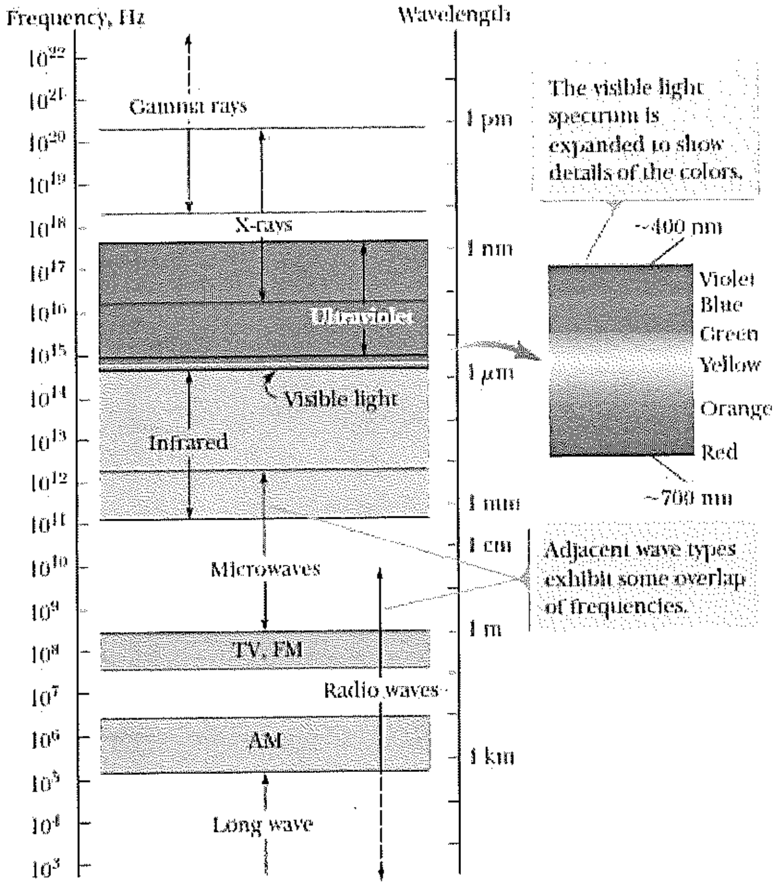
Wien's displacement law:

$$\lambda_{max} T = 2.898 \times 10^{-3}\ m \cdot K$$

Compton's effect:

$$\lambda' - \lambda_0 = \frac{h}{m_e c}(1 - cos\theta)$$

The electromagnetic spectrum



SECTION A

1. (a) What was the significance of the Davisson and Germer experiment?

[3]

(b) Lightning produces a maximum air temperature on the order of 10^4 K, whereas a nuclear explosion produces a temperature on the order of 10^7 K.

(i) Use Wien's displacement law to find the order of magnitude of the wavelength of the thermally produced photons radiated with greatest intensity by each of these sources.

[2]

(ii) Name the part of the electromagnetic spectrum where you would expect each to radiate most strongly.

[2]

(c) A π^0 meson is an unstable particle produced in high energy particle collisions. Its rest energy is approximately 135 MeV, and it exists for a lifetime of only 8.70×10^{-17} s before decaying into two gamma rays. Using the uncertainty principle, estimate the fractional uncertainty $\Delta m/m$ in its mass determination.

[4]

(d) Photons of wavelength 450 nm are incident on a metal. The most energetic electrons ejected from the metal are bent into a circular arc of radius 20.0 cm by a magnetic field with a magnitude of 2.00×10^{-5} T. What is the work function of the metal?

[4]

(e) The wave function for a quantum particle is given by

$$\psi(x) = \begin{cases} Ax & \text{for } 0 \le x \le 1.00 \\ 0 & \text{elsewhere} \end{cases}$$

Find the value of the normalization constant A, and the probability that the particle will be found between $x = 0.300$ and $x = 0.400$.

[5]

(f) The matrix below operates on the input data (y_{in}, V_{in}) to yield the output data (y_{out}, V_{out}) for a magnifying glass.

$$\mathbb{M} = \begin{bmatrix} 1 - q/f & p + q - pq/f \\ -1/f & 1 - p/f \end{bmatrix}$$

(i) Find the image location and magnification.

[2]

(ii) Describe the image in the limit $p \longrightarrow f$.

[3]

(g) An antelope is at a distance of 20.0 m from a converging lens of focal length 30.0 cm. The lens forms an image of the animal. If the antelope runs away from the lens at a speed of 5.00 m/s, how fast does the image move? Does the image move toward or away from the lens?

[5]

(h) A point source is located at a given position. You have a lens with focal length f. How far away from the source should you place the lens so that the distance from the source to the image is minimum?

[5]

(i) The input data (y_1, V_1) for a ray that propagates a distance L in space with constant index of refraction is modified by the transfer matrix $\Im$ to yield the output data (y_2, V_2). Find the transfer matrix $\Im$.

[5]

SECTION B

2. (a) Assuming that all angles are small with respect to the horizontal, derive the transfer matrix which acts on a ray of light crossing a flat interface between two materials with indices of refraction n_1 and n_2.

[5]

(b) An object is located a distance a to the left of a single flat interface between two materials with indices of refraction n_1 and n_2. A distance b to the right of this interface is a concave mirror with radius R. See Figure 1 below.

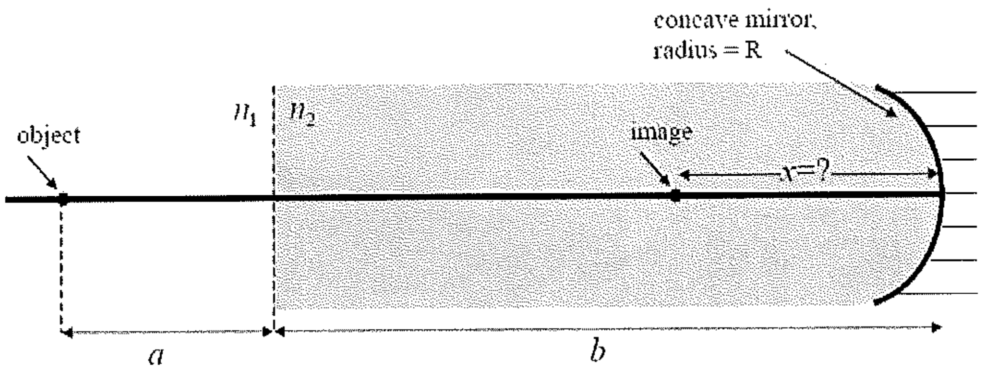


Figure 1

(i) Use the matrix method to find the location of the image (that is, the horizontal distance x from the mirror.)

[10]

(ii) Show that the condition that must be satisfied for a real image is

$$\frac{2}{R}\left(a + b\frac{n_1}{n_2}\right) > \frac{n_1}{n_2}$$

Check the limiting case $n_1 = n_2$.

[5]

3. The left hand of a long plastic rod of refractive index 1.56 is ground and polished to a convex (outward) spherical surface of radius 2.8 cm. An object 2 cm tall is located in the air and on the axis at a distance of 15 cm from the vertex. See Figure 2 below.

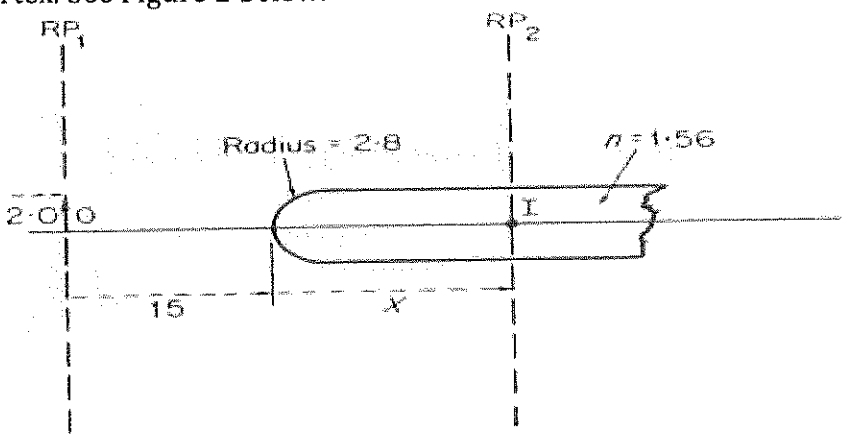


Figure 2

Find the position and size of the image inside the rod

[20]

4. A ray of light with input data (y_{in}, V_{in}) starts at a distance d_1 from a thin lens (of focal length f) and is observed a distance d_2 away from the lens on its right hand side with output data (y_{out}, V_{out}).

(a) Derive the ray transfer matrix which modifies (y_{in}, V_{in}) to (y_{out}, V_{out}).

[6]

(b) Show that the imaging condition leads to the lens formula

$$\frac{1}{d_1} + \frac{1}{d_2} = \frac{1}{f}$$

[4]

(c) Find the magnification. Is the image upright or inverted? Explain your answer.

[5]

(d) Describe the form of the image for the case of $d_1 < f$.

[5]

5. A photon having energy $E_0 = 0.880\ MeV$ is scattered by a free electron initially at rest such that the scattering angle of the scattered electron is equal to that of the scattered photon as shown in Figure 3.

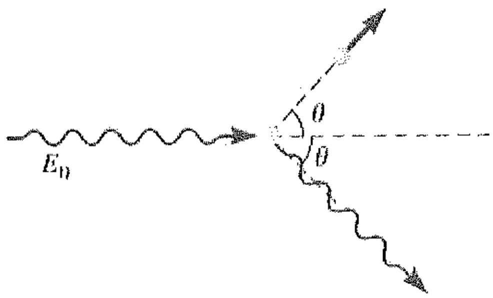


Figure 3

(a) Determine the scattering angle of the photon and the electron. [10]

(b) Determine the energy and momentum of the scattered photon. [5]

(c) Determine the kinetic energy and momentum of the scattered electron. [5]

6. (a) Since a charge e moving with an acceleration a in classical physics radiates energy at a rate of

$$\frac{dE}{dt} = -\frac{2}{3}\frac{e^2 a^2}{c^3}$$

the classical hydrogen atom is unstable, and the electron will spiral into the proton.

(i) Show from the above equation (assuming quasicircular motion) that the radius of the orbit decreases with time as

$$\frac{dr}{dt} = -\frac{4}{3}\frac{e^4}{m^2 c^3}\frac{1}{r^2}$$

Integrate this result and find the time t_0 it takes the electron to fall from an initial r_0 to $r = 0$. For $r_0 = \hbar^2/me^2 \approx 0.0529\ nm$ (the Bohr radius), what is this lifetime in seconds? [10]

(ii) What power would 1 gram of this "classical hydrogen" radiate initially, if each electron starts at one Bohr radius from the proton? [5]

(b) In his 1922 paper, F. S. Brackett reported a transition in hydrogen at a wavelength of $\lambda = 4.05 \pm 0.03\ \mu m$. Compare this with what you would expect for a transition from the $n = 5$ to $n = 4$ levels of the Bohr atom, given by the energy level formula

$$E_n = -\frac{mc^2\alpha^2}{2n^2}$$

[5]

$[\alpha = 1/137.036 \quad ; \qquad mc^2 = 511\ keV; \quad e^2 = \hbar c\alpha]$

END OF PAPER