BINDURA UNIVERSITY OF SCIENCE EDUCATION
BACHELOR OF SCIENCE EDUCATION DEGREE
PHYSICS PART 2
PH202: QUANTUM PHYSICS 1
DURATION: 3 HOURS

INSTRUCTIONS:
Answer ALL parts of Section A and any THREE questions from Section B.
Section A carries 40 marks and Section B carries 60 marks.

Electron charge,	$e = 1.60 \times 10^{-19}\,C$
Planck's constant,	$h = 6.63 \times 10^{-34}\,Js$
Mass of an electron,	$m_e = 9.11\,x\,10^{-31}\,kg$
Acceleration due to gravity,	$g = 9.81\,ms^{-2}$
Permittivity of free space,	$\varepsilon_0 = 8.85 \times 10^{-12}\,Fm^{-1}$
Speed of light,	$c = 3 \times 10^{8}\,ms^{-1}$

SECTION A

1. (a) List the five postulates of quantum mechanics [5]

(b) Derive the Compton shift equation $\Delta\lambda = \lambda' - \lambda = \frac{h}{m_e c}(1 - \cos\theta)$. [4]

(c) Find the first excited state of the harmonic oscillator. [4]

(d) An electron is bound to a region of space by a spring like force with an effective spring constant of $k = 95.7eV/nm^2$.

(i) What is its ground-state energy? [3]

(i) How much energy must be absorbed for the electron to jump from the ground state to the second excited state? [3]

(e) Consider a particle whose normalized wave function is

$$\psi(x) = 2\alpha\sqrt{\alpha}\,xe^{-\alpha x} \qquad x > 0$$
$$= 0 \qquad x < 0$$

Calculate (i) $\langle x \rangle$ [3]

(ii) $\langle x^2 \rangle$ [2]

(f) A particle moving in one dimension interacts with a potential *V (x)*

In a stationary State of this system

show that

$$\frac{1}{2}\left\langle x\frac{\partial}{\partial x}V\right\rangle = \langle T\rangle$$

Where *T* = *p2/ 2m* is the kinetic energy of the particle [3]

(g) Show that

$$\frac{d}{dt}\langle A\rangle = 0$$

[3]

(h) State *Ehrenfest*'s principle [2]

(i) A particle of mass *m* has the wave function

$$\Psi(x,t) = Ae^{-a[(mx^2/\hbar)+it]}$$

where *A* and *a* are positive real constants.

(i) Find *A*. [2]

(ii) For what potential energy function, $V(x)$ is this a solution to the Schrödinger equation? [3]

(ii) Calculate the expectation values of x^2 [3]

SECTION B

2(a) Give a physical reason why we require observables to be represented by Hermition operators. [5]

(b) Find the first excited state of the harmonic oscillator. [9]

(c) If $\hat{A}$ is Hermitian, show that

$$\langle\hat{A}^2\rangle \geq 0$$

[6]

l has the initial wave function

$$\Psi(x,0) = \begin{cases} Ax, & 0 \le x \le a/2, \\ A(a-x), & a/2 \le x \le a. \end{cases}$$

(i) Sketch $\Psi(x,t)$, and determine the constant A. [3]

(ii) Find $\Psi(x,t)$. [4]

(iii)What is the probability that a measurement of the energy would yield the value ? [3]

3(b) A Solution to the Schrödinger Equation Show that for a free particle of mass m moving in one dimension, the function $\psi(x) = A\sin kx + B\cos kx$ is a solution to the time-independent Schrodinger equation for any values of the constants A and B. [5]

(c) Given a one-dimensional box of width 1mm.What value of n corresponds to a state of energy 0.01eV [5]

4 (a) Use the WKB approximation to determine the bound-state energies of the potential well

$$V(x) = \frac{V_0}{a}|x|, \qquad |x| \le a$$

$$V(x) = V_0 = \frac{1}{m}\left(\frac{h}{a}\right)^2, \qquad |x| > a$$

[10]

(b) A uniform homogeneous beam of electrons is incident on a rectangular potential barrier of height V . Each electron in the beam has energy $E > V$ and unit amplitude wave function

$$\varphi_{\text{inc}} = e^{ik_1 x}$$

If the transmitted electrons have wave function

$$\varphi_{\text{trans}} = \varphi_{\text{III}} = 0.97e^{ik_1 x}$$

(i) What is the total wave function φ_{I}, , of electrons in region 1? [4]

(ii) If E=10eV and V=5eV , what is the minimum barrier width compatible with the information given
[6]

5(a) (i) what are the values of k and K at the "turning points" of a potential hill or potential barrier? [5]

(ii) What are the values of WKB wave functions, $|\varphi_{\mathrm{I}}|, |\varphi_{\mathrm{II}}|, |\varphi_{\mathrm{III}}|,$ I, at these points (for either bound or unbound states)? [5]

(iii) A student argues the following: We see from part (ii) of this problem that WKB

Wave functions blow up at the turning points and are therefore invalid. Such wave functions cannot be of any use. Is the student correct?

Explain. [5]

(b). Given that

$$\langle x|\hat{p}|x'\rangle = -i\hbar\frac{\partial}{\partial x}\delta(x-x')$$

Show that for free-particle motion

$$\langle x|e^{-it\hat{H}(\hat{x},\hat{p})/\hbar}|x'\rangle = \left[\exp -\frac{it}{\hbar}H\left(x, -i\hbar\frac{\partial}{\partial x}\right)\right]\delta(x-x')$$

[5]

6 (a) The Heisenberg uncertainty is often viewed as a limitation to measurements, but in combination with energy minimization it also has significant power in predicting the magnitude of quantum mechanical effects. Assume that you have a particle of mass *m* in a harmonic potential

$$V(x) = \frac{1}{2}m\omega^2 x^2.$$

(i) Argue in one or two sentences why a particle localized in a very narrow region around $x = 0$ would have a large total energy. [4]

(ii) By minimizing the total energy subject to the Heisenberg uncertainty relation between position spread Δx and momentum spread Δp the particle, calculate the size Δx of the ground state in the harmonic potential [6] b Show that in the nth eigenstate of the harmonic oscillator, the average kinetic energy (T) is equal to the average potential energy *(V)* (the virial theorem) . That is,

$$\langle V\rangle = \frac{K}{2}\langle x^2\rangle = \langle T\rangle = \frac{1}{2m}\langle p^2\rangle = \frac{1}{2}\langle E\rangle = \frac{\hbar\omega_0}{2}\left(n+\frac{1}{2}\right)$$

[10]

END OF EXAM PAPER